* Motion with constant acceleration.

- $\vec{a}$ is constant both in magnitude and direction
- $\vec{a}_{ave}$ = average acceleration, $\vec{a}_{inst.}$ = instantaneous acceleration

if $\vec{a}$ is constant, then $\vec{a}_{ave} = \vec{a}_{inst.} = \vec{a}$.

$$\Rightarrow \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} = \vec{a}\Delta t \Rightarrow \vec{v}_f - \vec{v}_i = \vec{a}\Delta t$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t \quad \text{——— (1)}$$

Thus, with constant $\vec{a}$, $\vec{v}$ is <u>always</u> a linear function of t.

Thus, we can express $\vec{v}_{ave}$ in any time interval as the average (or arithmatic mean) of the initial velocity $\vec{v}_i$ and the final velocity $\vec{v}_f$:

$$\vec{v}_{ave} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right) \text{ — (2)}.$$ This eq is valid <u>only</u> in such case: when $\vec{a}$ is constant.

Now, $\vec{v}_{ave} = \frac{\Delta \vec{x}}{\Delta t} \Rightarrow \Delta \vec{x} = \vec{v}_{ave} * \Delta t$ ~~~~ (*)

- plug $\vec{v}_f$ from (1) into (*), $\Delta \vec{x} = \left[\frac{\vec{v}_f + \vec{v}_i}{2}\right]\Delta t$ ~ (*)

$$\Delta \vec{x} = \left[\frac{\vec{v}_i + \vec{a}\Delta t + \vec{v}_i}{2}\right]\Delta t \Rightarrow \Delta \vec{x} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2 \quad \text{— (3)}$$

($t \equiv \Delta t$)

- plug Δt from (1) into (*) $\rightarrow \Delta t = \left|\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right|$, then

$$\Delta \vec{x} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\left(\frac{\vec{v}_f - \vec{v}_i}{\vec{a}}\right) \Rightarrow 2\vec{a}\Delta \vec{x} = v_f^2 - v_i^2$$

$$\therefore v_f^2 = v_i^2 + 2\vec{a}\Delta \vec{x} \quad \text{——— (4)}$$

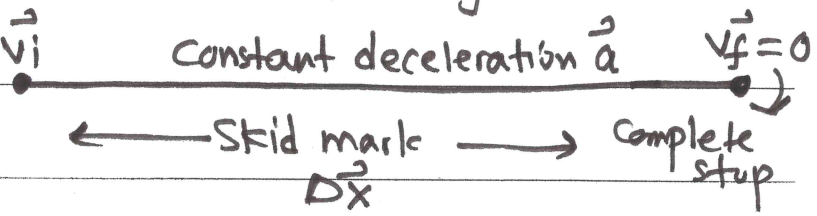
* Eq ③: with constant $\vec{a}$, the position $\Delta\vec{x}$ is always a quadratic function of t: a parabola curve.

* One needs to bear in mind that all the parameters in the 4 eqs are vectors (±ve signs) but t and $\vec{v}^2$.

* Eq ①: if $\vec{a}=0$, then $\vec{v}_f=\vec{v}_i$ ✓

Eq ③: if $\vec{a}=0$, then $\Delta\vec{x}=\vec{v}_i t$, but $\vec{v}_i=\vec{v}_f=\vec{v}$, then $\Delta\vec{x}=\vec{v}t$ ⇝ Hmm....: Ms. Dalal, 4th grade science !!

■ Problem 2.28:



Given: $\vec{a}, \Delta\vec{x}, \vec{v}_f$

Calculate $\vec{v}_i$? ➡ Look for an eq that does not contain t.

$v_f^2 = v_i^2 + 2\vec{a}\Delta\vec{x} \rightarrow 0 = v_i^2 + 2[-a][+\Delta x]$ ✓

■ Past exam: A box of mass 2kg is moving with a velocity $+20\,ms^{-1}$ when it is located at $x=+10m$. Its location 5 sec later becomes $x=-30\,m$. What is the acceleration of the particle? ⇒ $\Delta\vec{x}=\vec{v}_i t+\frac{\vec{a}}{2}t^2$

$\vec{x}_f-\vec{x}_i=\vec{v}_i t+\vec{a}t^2/2 \Rightarrow -30-10=+20\times5+\frac{25}{2}\vec{a}$,

solve for $\vec{a}=-11.2\,ms^{-2}$.

■ Recall: Lecture 2 page 4: $x(t)=3-4t+2t^2$ ⇒ find the one-to-one correspondence between x(t) and eq ③.

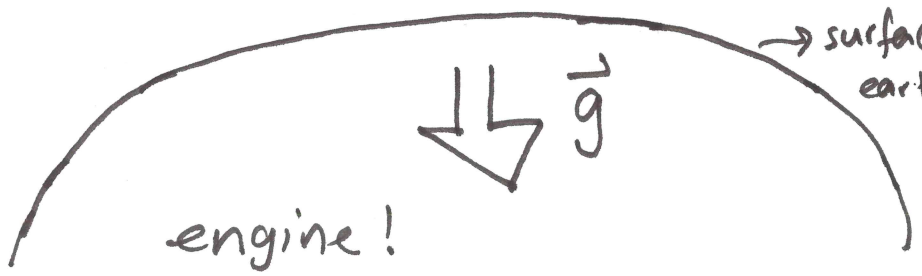
Freely falling objects:

If an object is moving freely under the influence of gravity alone, then it is considered to be freely falling object, regardless of its initial condition, whether it is thrown upward or downward or released from rest. ⇒ Air resistance is negligible.

$\vec{a}$ = the acceleration of gravity = $\vec{g}$.

$\vec{g} = 9.8\ m/s^2$ downward ⇒ $\vec{g}$ is constant, thus the motion of a freely falling object is equivalent to the motion of a particle under constant acceleration in one dimension (vertically). Therefore, the 4 kinematic eqs we just developed are valid and can be applied. Let $\vec{a} = \vec{g}$ and $\Delta\vec{x}$ becomes $\Delta\vec{y}$.

* Please keep in mind that $\vec{g}$ is ALWAYS pointing downward and its value is constant ($9.8\ m/s^2$), regardless of the motion of the object!



■ Past exam: A box is dropped from the top of a building. It takes (t sec) to hit the ground. What is the height of the building?

Given: $\vec{v}i = 0$, $\vec{a} = \vec{g}$, t of the flight ✓ $\vec{y}$?

$\therefore \vec{y} = \vec{v}it + \vec{a}t^2/2 \Rightarrow \vec{y} = 0 + \frac{-gt^2}{2} = -ve\ m.$

rational: $\vec{y}$ is a displacement so it is expected to be -ve. The height of the building is just $\frac{+gt^2}{2}$ m.

Plz: do example 2.10.

■ Past exam: A box is released from 100 m high building. The magnitude of the average velocity during the first 3 sec of its fall is:

Eq (2): $\vec{v}_{ave} = \frac{\vec{v}i + \vec{v}f}{2} = \frac{\vec{v}f}{2} = \frac{\vec{v}i + \vec{a}t}{2} = \frac{0 - gt}{2} = \frac{-gt}{2}$

$\therefore$ magnitude $\Rightarrow \left|\frac{-gt}{2}\right| = +1.5 * g = +14.7\ m s^{-1}$

another approach: $\vec{v}_{ave} = \frac{\Delta\vec{y}}{\Delta t} = \frac{\vec{v}it + \vec{a}t^2/2}{t} = \frac{0 - gt^2/2}{t}$ ✓

• Let's validate our answer: how far will the box have fallen after t = 3 sec? $\vec{y} = \frac{-gt^2}{2} = -44.1\ m$, so the box is still in the air.

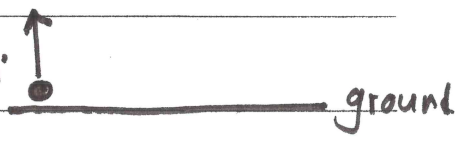
Past exam: A stone is thrown from the top of a building downward with a speed of $+5\,m\,s^{-1}$. It hits the ground with a speed of $+34\,m\,s^{-1}$. The height of the building is:

$$v_f^2 = v_i^2 + 2\vec{a}\Delta y \Rightarrow v_f^2 = v_i^2 + 2(-g)\Delta\vec{y} \Rightarrow$$

$$\Delta\vec{y} = \frac{v_f^2 - v_i^2}{-2g} = -ve \checkmark \Rightarrow \text{height is } | \; | = 58\,m.$$

$\Rightarrow$ Do example 2.11.

Example: • is thrown upward with initial speed v_i.



- t_{peak}: time it takes for • to reach the maximum height.

$\Rightarrow$ At the max height, • is momentarily at rest $\Rightarrow \vec{v_f} = 0$

$$\therefore \vec{v_f} = \vec{v_i} + \vec{a}\,t_{peak} \rightarrow 0 = +v_i - g\,t_{peak} \rightarrow \boxed{t_{peak} = \frac{v_i}{g}}$$

- y_{max} = the max height • goes:

$$\vec{y}_{max} = \vec{v_i}\,t_{peak} - \frac{g}{2}t_{peak}^2 = \frac{v_i^2}{g} - \frac{g}{2}\left(\frac{v_i}{g}\right)^2 = \boxed{+\frac{v_i^2}{2g}}$$

- t_{return}: time needed for • to return from y_{max} to the ground

Hmm...

dimensional analysis: is it $\frac{v_i}{g}$ or $\frac{g}{v_i}$?

$\vec{y} = \vec{v_i}t + \frac{at^2}{2} \Rightarrow \vec{y} = -y_{max} \Rightarrow -y_{max} = 0 - \frac{gt^2_{return}}{2}$

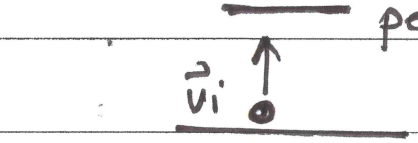
$\therefore t^2_{return} = +\frac{2 * y_{max}}{g} \Rightarrow \boxed{t_{return} = \frac{v_i}{g}}$

$\Rightarrow$ Freely falling: Symmetry : t_{peak} ↑ • ↓ $= t_{return}$

- t_{flight} : the time • is in the air before it comes back to the ground $= t_{peak} + t_{return} = \frac{2v_i}{g}$.

Plz: do examples 2.12, 2.13, 2.15 and problems 2.39, 2.43

- Example: • has an upward velocity of 25 m/s when it reaches one fourth of its maximum height. What is the initial speed v_i?



when $y = \frac{1}{4} y_{max}$, $v = 25 \Rightarrow$ solve for v_i

$v_f^2 = v_i^2 - 2g\left(\frac{y_{max}}{4}\right)$, plug in $y_{max} = \frac{v_i^2}{2g} \Rightarrow$

$v_i^2 = \frac{4}{3} v_f^2 \Rightarrow v_i = \sqrt{\frac{4}{3}} * 25$ ✓ $v_i > v_f$ ✓.

- Past exam: A particle dropped from height h takes time t to hit the ground. If the particle is dropped from height 2h, the time it takes to hit the ground is:

$[2t, \frac{t}{\sqrt{2}}, \frac{t}{2}, \sqrt{2}t, 4t]$

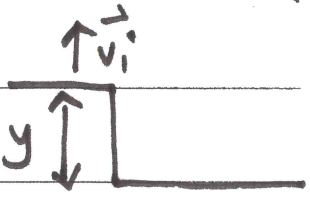
✓

■ Past exam: A stone is thrown from 30m high building upward with a speed of $20\,ms^{-1}$. What is the time needed for the stone to strike the ground?

[2.0 s , 3.2 s , 5.2 s , 1.2 s , 4.1 s]

ثمّة أسلوب أسلم (لأغراض الامتحان) وثمة أسلوب أفهم (لأغراض التعلّم)! ⇐

(I) $t_{flight} = t_{peak} + t_{return}$



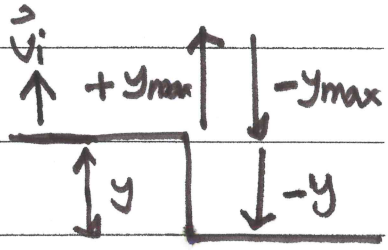
- t_{peak} (recall our results in page 5) $= \frac{v_i}{g}$
- t_{return}: asymmetry! $\Rightarrow y_{max} = v_i^2/2g$, then

$$-(y + y_{max}) = 0 - \frac{g}{2}t_{return}^2 \Rightarrow t^2 = +\frac{2(y+y_{max})}{g}$$

$$\therefore t_{return}^2 = \frac{2y}{g} + \frac{2y_{max}}{g} = \frac{2y}{g} + \frac{v_i^2}{g^2}$$

$$\therefore t_{flight} = \frac{v_i}{g} + \sqrt{\frac{2y}{g} + \left(\frac{v_i}{g}\right)^2} \cong 2.0 + 3.2 = 5.2\,sec.$$

(II) $\Delta\vec{y} = \vec{v_i}t + \vec{a}t^2/2$.



$\Delta\vec{y} \equiv$ displacement $= +y_{max} + (-y_{max}) + (-y)$

$$\therefore -y = \oplus v_i t - gt^2/2$$

$$\frac{gt^2}{2} - v_i t - y = 0 \Longleftrightarrow \text{solve the quadratic eq for } t$$

brush-up: $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore t_{flight} = \frac{v_i}{g} \pm \sqrt{\left(\frac{v_i}{g}\right)^2 + \frac{2y}{g}}$$, you pick the + term!

- quiz: the 4th option, $t = 1.2$ sec corresponds to what?

$$-30 = \ominus 20t - \frac{gt^2}{2} \rightarrow t = 1.165 \simeq 1.2\ !!$$

7/7