

Chapter 9: Static equilibrium

- Static equilibrium is defined as mechanical equilibrium for the special case of an object at rest: it has no translational or rotational speed.
- An object can be in static equilibrium only if the net force acting on it is zero and the net torque acting on it is zero.
- It is important to remember that torque is always defined with respect to a pivot point, as discussed in Lecture 10. If we are solving a static equilibrium problem, the net torque has to be zero for any pivot point chosen. Thus, we have the freedom to select a pivot point that best suits our purpose!
- ⇒ A clever selection of a pivot point is often the key to a quick solution. For example, if an unknown force is present in the problem, we can select the point where the force acts as the pivot point. Then, that force

will not enter into the torque equation because it has a lever arm of length zero (recall lecture 10, page 3).

• We can formulate the two necessary conditions for equilibrium of an object:

$$\sum \vec{F}_{\text{ext}} = 0 \quad \text{--- (1) and}$$

$$\sum \vec{\tau}_{\text{ext}} = 0 \quad \text{--- (2) .}$$

Note that these 2 vector expressions are equivalent, in general, to 6 scalar equations:

3 from the first condition and 3 from the second, corresponding to the x , y and z components.

In this chapter, we do restrict our analysis with problems in which all the forces lie in the xy plane.

• With this restriction, we must deal with only:

Quiz: [6 - "1" = 5 or 4 or 3] scalar eqs.?

Can you politely ask your 2nd grade math teacher?

• 2 scalar equations come from balancing the forces in the x and y directions.

• 1 scalar equation comes from balancing the torque about the z-axis through any pivot "point" in the xy plane.

⇒ Thus, the 3 equilibrium equations are:

$$\sum F_x = \sum F_y = \sum \tau_z = 0 \quad \text{--- (3)}$$

Remember again that the choice of the axis of rotation (the pivot) is arbitrary. Usually, the most convenient axis for calculating torques is one through a pivot at which several forces act, so their torques around this axis are zero!

• Try to guess, on a hunch, the correct direction for any force that is not specified. If this leads to a -ve sign in your solution for that force, do not panic!! it merely means that the direction of the force is the opposite of what you guessed.

ee

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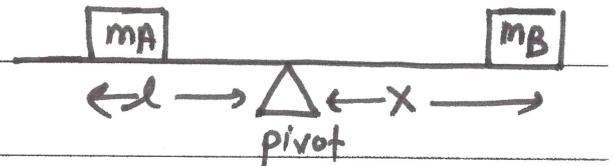
Examples involving static equilibrium:

The three conditions, eq (3), are all we need to solve a very large class of problems involving static equilibrium. To solve these problems, we use only algebra and trigonometry (7th - 8th grades!).

Let's start with the seesaw example, lecture 10, page 7, for which the answer seems obvious. This will provide practice with the method and show that it leads to the right answer.

Example 9.4: Seesaw:-

$m_A, m_B, M(\text{board}),$ and l



are given; at what distance

(x) from the pivot must m_B be placed to balance the seesaw?

⇒ The selection of the proper pivot point can make our computations simple. For a seesaw, the natural selection is at the board's pivot itself!

$$\circ \circ + m_A g l + (-m_B g x) = 0 \Rightarrow x = \left(\frac{m_A}{m_B} \right) l$$

⇒ As reported in the example, $m_A > m_B$, thus $x > l$.

This is just the common sense!

If $m_A = m_B \rightarrow x = l$ ✓

• We have stressed the fact that we can use any pivot point. Thus, if we change the pivot point, it can make the calculations more complicated in certain situations, but the end result of the calculation will not change.

⇒ Suppose that the pivot for the seesaw is placed instead at m_A . Let's verify that this leads to the same result.

$$n l = M g l + m_B g (x + l),$$

$$\text{but } n = (m_A + m_B + M) g$$

$$\Rightarrow \text{solve for } x \Rightarrow x = \left(\frac{m_A}{m_B} \right) l \checkmark$$

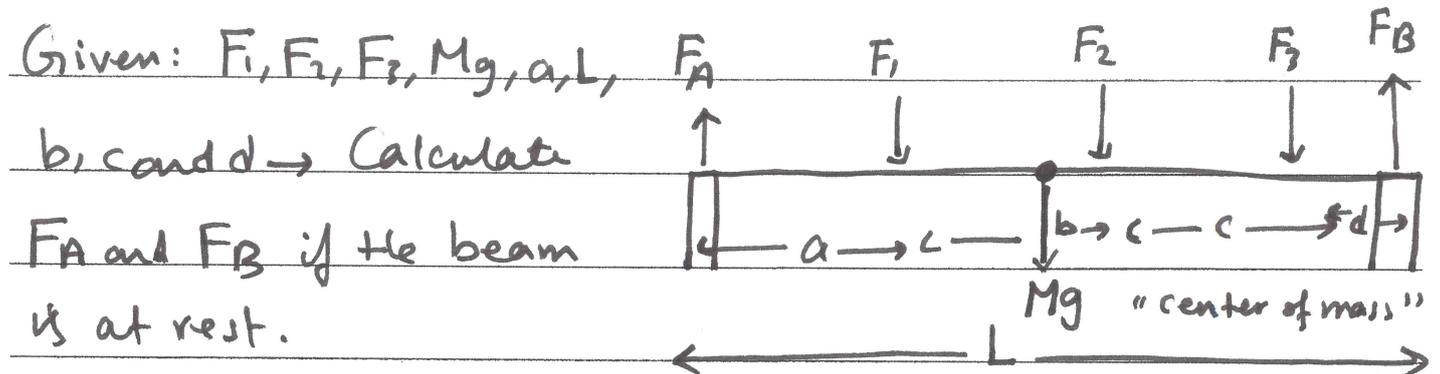
• Quiz: How big does m_A need to be to balance m_B if $x = 3l$, that is, if m_A is three times closer to the pivot than m_B ?

$$\left[m_A = \sqrt{3} m_B, m_A = 3 m_B, m_A = 9 m_B \right] ?$$

• Do examples 9.2, 9.3 and problem 9.12 = 9.11 !

7th edition 6th edition

2] Problem 9.16 (7th) \equiv 9.15 (6th)



\Rightarrow Right off: $F_A + F_B = F_1 + F_2 + F_3 + Mg$ — (1)

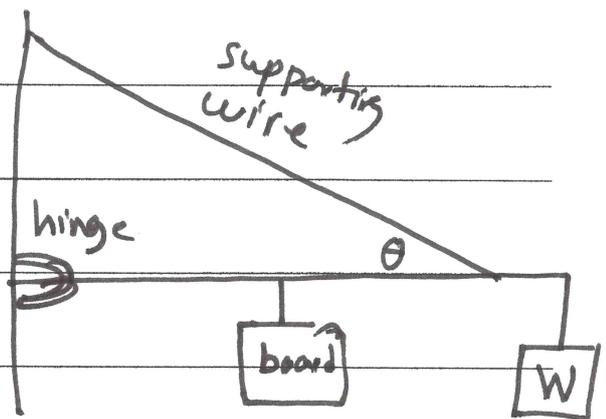
one eq with 2 unknowns. \Rightarrow choose the pivot, let's say about F_B

$F_A(a+b+c+d) = F_1(b+c+d) + F_2(c+d) + F_3(d) + Mg(\frac{L}{2})$ — (2)

3] Problem 9.18 (7th) \equiv 9.20 (6th)

\Rightarrow Given: W , weight of board and θ .

\Rightarrow Calculate the tension in the supporting wire and the force exerted by the hinge on the beam.



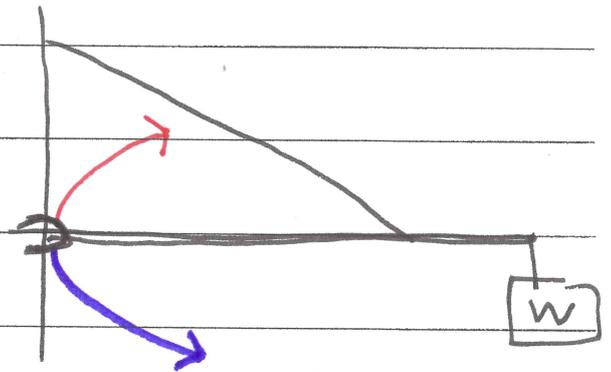
\Rightarrow Hmm... How to assign the

direction of the hinge's force? [Recall last paragraph].

\Rightarrow It might be helpful to imagine what would happen if

this force were suddenly removed. Therefore, if the hinge (and the wall) were to vanish suddenly, the left end of the board would move to the left as it begins to fall:

⇒ This scenario tells us that the hinge is not only holding the board up but is also pressing outward



against it. Therefore, we draw the vector \vec{F}_{hinge} in the direction shown:

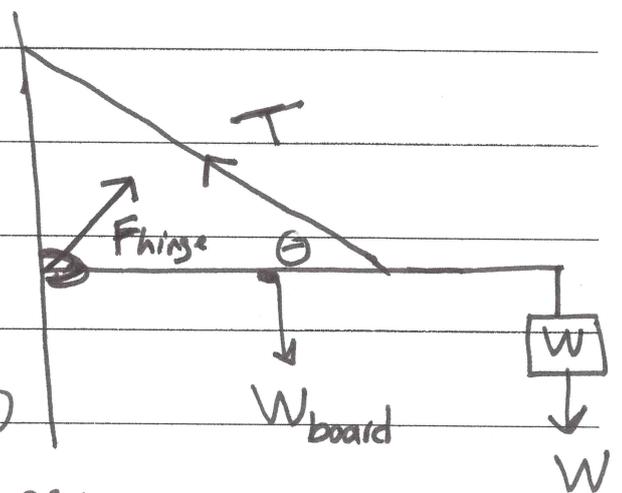


⇒ The forces acting are:

$$F_{\text{hinge}} = T \cos \theta \quad \text{--- (1)}$$

$$F_{\text{hinge}} + T \sin \theta = W + W_{\text{board}} \quad \text{--- (2)}$$

⇒ 3 unknowns ⇒ need one more eq. :-



It seems natural to pick the pivot at the hinge itself.

This has the advantage that we do not need to pay attention to \vec{F}_{hinge} , because it has a lever arm of zero and consequently make no contribution to the torque.

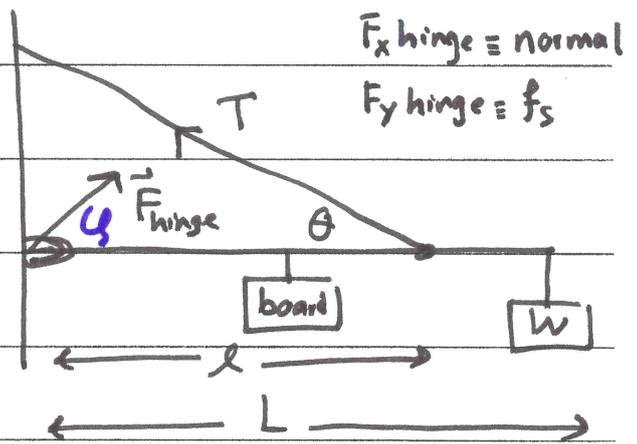
⇒ With l and L , the torque equation reads:-

$$(T \sin \theta) * l = W * L + W_{\text{board}} * \frac{L}{2}$$

Once we have found T ,

we can determine

$F_{x \text{ hinge}}$ and $F_{y \text{ hinge}}$.



Validation #1: Let ϕ be the angle of \vec{F}_{hinge} relative to the board. Calculate $\phi = \tan^{-1} \left(\frac{F_{y \text{ hinge}}}{F_{x \text{ hinge}}} \right)$. If ϕ is acute (measure less than 90°), then this indicates that our estimate of the direction of \vec{F}_{hinge} was accurate.

Validation #2: had we selected some other pivot for the torque eq, the solution might differ in the

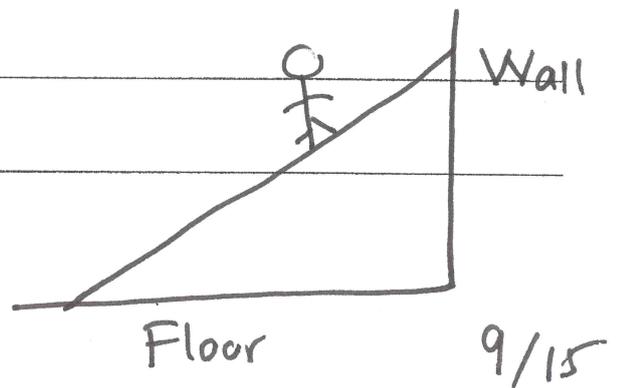
details but the answers would be the same. Choose, if you don't mind!, the pivot at the center of the board (its weight), then the torque τ would involve both \vec{T} and \vec{F}_{hinge} . This τ , coupled with τ ① and ② in page 7, however, could still be solved for the unknowns. Enjoy!!

Do Example 9.6

4 A PHY 105 student standing on a ladder:

- Your text has used a "comparative" adjective while discussing Example 9.7: A "more" difficult example! the ladder.
- Question 3 in [Search and Learn] in your 7th edition text or in Problem 9.27 in your 6th edition is a "bit" more difficult!
- That being said, and without any exaggeration, you might need to use a "superlative" construction to express problem 4! You'd better watch out!!!

⇒ The ladder rests on a rough floor and leans against a smooth wall.



- The length and the mass of the ladder: l, m_1 .
- The angle of the ladder relative to the floor: θ .
- The PHY 105 standing on the ladder: the mass is m and he/she stands on a rung that is \underline{r} along the ladder measured from where the ladder touches the floor.

(i) What friction force must act on the bottom of the ladder to keep it from slipping? Neglect the small friction force between the smooth wall and the ladder.

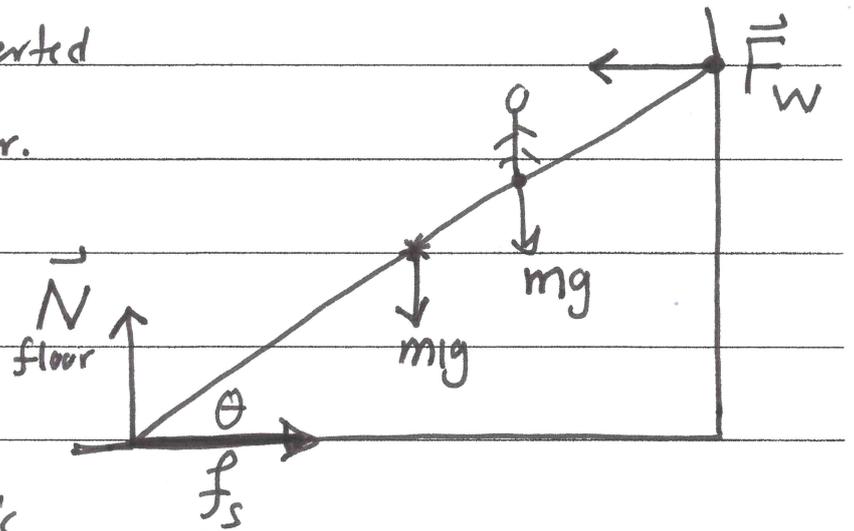
$\Rightarrow \vec{F}_w =$ normal force exerted by the wall on the ladder.

$\Rightarrow \vec{N}_{\text{floor}} =$ normal force exerted by the floor on the ladder.

$\Rightarrow \vec{f}_s =$ the force of static

friction between the floor and the bottom of the ladder.

Note that \vec{f}_s is directed to the right: if the ladder



Slips, its bottom will slide to the left, and the friction force must necessarily oppose that motion.

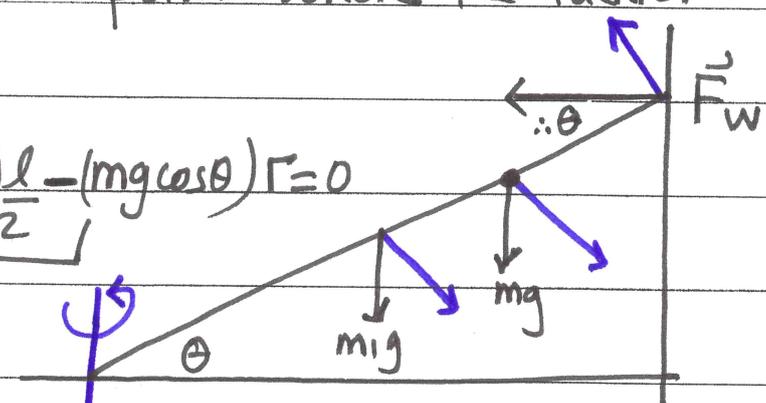
As instructed, we neglect the force of friction between wall and ladder. [hazard a guess about its direction?]

$$\Rightarrow f_s = F_w \quad \text{--- (1)}$$

$$\Rightarrow N = (m_1 + m)g \quad \text{--- (2) } \checkmark \text{ (known).}$$

\Rightarrow We neglect the wall friction force, which would otherwise have come in here! • Hmm...

\Rightarrow Take the pivot at the point where the ladder touches the floor.

$$+(F_w \sin \theta)l - (m_1 g \cos \theta) \frac{l}{2} - (mg \cos \theta) r = 0$$


$$\text{Solve for } F_w = \left[\frac{1}{2} * m_1 g + \frac{r}{l} * mg \right] \cot \theta \quad \text{--- (3)}$$

• However, we already found from (1) that $F_w = f_s$ ✓

• In your text $\frac{r}{l} = 0.7$ ✓

(search and learn) $\frac{r}{l}$
question 3

(ii) Suppose that the coefficient of static friction between the ladder and the floor is μ_s .

Will the ladder slip?

\Rightarrow The maximum static friction $= \mu_s * N$.

We found N (eq 1). So f_{smax} is directly determined. Thus, ~~the~~ the ladder will not slip as long as the force from the wall, F_w , is smaller than f_{smax} , leading to the static equilibrium condition:

$$\left[\frac{l}{2} * mg + \frac{r}{e} * mg \right] \cot \theta \leq \mu_s (m_1 + m) g \quad \text{--- (4)}$$

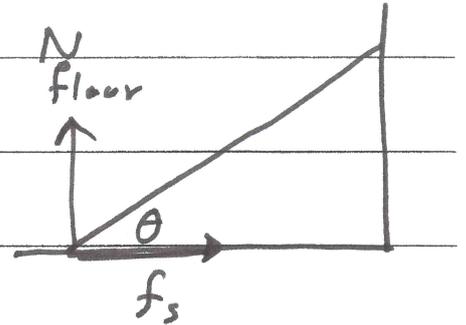
$$\Rightarrow \left(\frac{m_1}{2} + \frac{r m}{e} \right) \cot \theta \leq \mu_s (m_1 + m) \quad \checkmark \quad \text{--- (4)*}$$

(iii) What happens as the PHY 105 student climbs higher on the ladder?

From eq (4), we see that F_w grows larger with increasing r . Eventually, this force will overcome f_{smax} , and the ladder will slip. You can now understand why it's not a good idea to climb too high on a ladder in this kind of situation.

(iv) In which direction does the force that the floor exerts on the ladder point?

The floor exerts a force \vec{F}_{floor} on the ladder. The two components of this force are f_s and N_{floor} .



Calling the angle between this force and the floor ϕ , we have $\phi = \tan^{-1} \left[\frac{F_y}{F_x} \right] = \tan^{-1} \left[\frac{N_{\text{floor}}}{f_s} \right]$.

The problem statement says that the ladder makes an angle θ with the floor. Note that θ is not the same as ϕ ; the force exerted by the floor on a ladder in a situation like this does not, in general, point along the ladder! The ladder is rigid and not flexible like a string or cable: see the direction of \vec{F}_c in example 9.7 and in question 3 - search and learn.

(v) Suppose the ladder is of length $l = 3.04 \text{ m}$, with mass $m_1 = 13.3 \text{ kg}$, rest on a rough floor at an angle of $\theta = 65.2^\circ$. The PHY 105 student, who has a

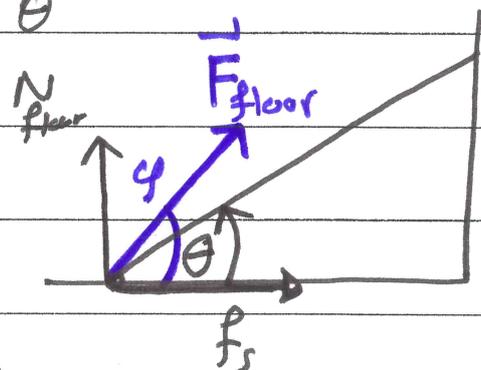
mass of $m = 62.0 \text{ kg}$, stands on a rung that is $r = 1.43 \text{ m}$ along the ladder measured from where the ladder touches the floor. Now calculate the items (i), (ii) and (iv) and report numerical values. Verify that you should get the following results: -

For item (i) $F_w \approx 162.2 \text{ N}$.

For item (ii) with $\mu_s = 0.31$, $f_s \text{ max} \approx 228.8 \text{ N}$. So $f_s \text{ max}$ is well above the 162.2 N that we just found necessary for static equilibrium. In other words, the ladder will not slip.

For item (iv): $(\phi) = \tan^{-1}\left(\frac{N_{\text{floor}}}{f_s}\right) = \tan^{-1}\left(\frac{738}{162.2}\right)$
 $(\phi) = 77.6^\circ$. Note that $(\phi) \neq \theta$

(vi) Quiz: A ladder l rests on a rough floor and leans against smooth wall as shown:



Given: l = length of ladder, m = mass of ladder, μ_s between the ladder and the floor. Show that when the ladder is on the verge of slipping,

$\theta = \tan^{-1}\left[\frac{1}{2\mu_s}\right]$: that is θ does not depend on m or l of the ladder!

5 Muscles and Joints problems!

If you are planning to be an Orthopedic surgeon, then you'd better ponder these problems:-

- (i) Do example 8.8 (chapter 8 - section 4).
- (ii) Do example 9.8 (Chapter 9 - section 3).
- (iii) Do example 9.9 : bear in mind the fact that we do not need calculus to solve such problem; all the calculations use only the 7th-grade algebra and the 8th-grade trigonometry! I wish you'd stop nagging!
- (iv) Do problem 9.32 (7th = 6th).
 $F_M = 250\text{ N}$, $F_J = 243.6\text{ N}$
- (v) Do problem 9.33 (7th = 6th):
It is expected that F_M would be larger than its value in problem 9.32.