Chapter 9: Static Equilibrium

Lecture 1

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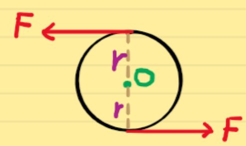
9-1] Conditions for equilibrium.

A wheel is fixed at point 'O' and can rotate about an axis that passes through 'O' and perpendicular to the page.



What is the net force acting on the wheel?

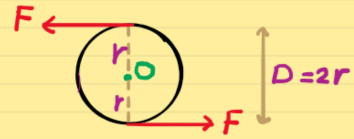
$\xrightarrow{} +\ \Sigma F = F - F = 0$. (can write $F_{net} = 0$ also).

Does the wheel move translationaly? (i.e to the right or to the left?) NO.

Does the wheel rotate about point O?

To answer this find the net torque about point O.

$\overset{+}{\circlearrowleft}$ $\tau_{net} = rF + rF = 2rF$



$\therefore$ Wheel rotates counterclockwise

$\Rightarrow$ it is NOT in static equilibrium.

No translational motion but it has rotational motion. Since $\tau_{net} \neq 0$.

How can we apply the two forces such that the wheel does NOT rotate?

$\Sigma\tau \equiv \tau_{net} = 0 \Rightarrow$ No rotational motion.

$\Sigma F \equiv F_{net} = 0 \Rightarrow$ No translational motion.

$\Rightarrow$ Wheel is in static equilibrium.

Conditions for static equilibrium

i) $\Sigma\tau = 0$ } Both must be

ii) $\Sigma F = 0$ } satisfied simultaneously.

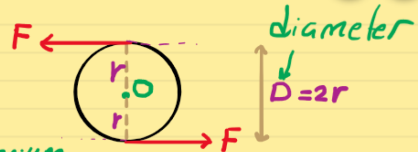
Couple

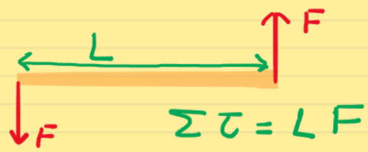
$\sum F = 0$ but $\sum \tau \neq 0$

$\therefore$ Wheel is Not in static equilibrium

Note above we found:

$\sum \tau = 2rF = DF$





Although the net force on the wheel is zero, the wheel will move (rotate). A pair of equal forces acting in opposite directions but at different points on an object (as shown above) is referred to as a couple.

A lever (العتلة)

The bar in Figure is being used as a lever to pry up a large rock. The small rock acts as a fulcrum (pivot point). The force required at the long end of the bar can be quite a bit smaller than the rock's weight mg, since it is the torques that balance in the rotation about the fulcrum.

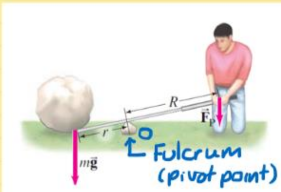
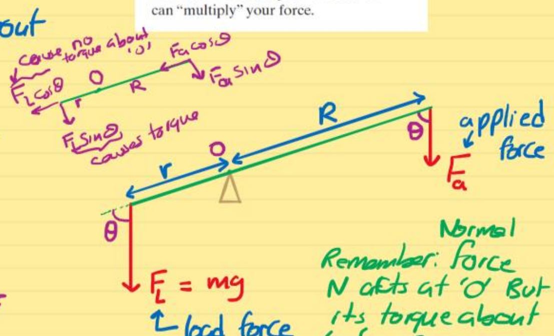


FIGURE 9–6 Example 9–3. A lever can "multiply" your force.

Find the net torque about the pivot point 'O'.



F_a acts to rotate the lever clockwise ⇒ Pry up the big stone (Load).

F_L acts to rotate the lever counterclockwise.

Remember: Normal force N acts at 'O' But its torque about 'O' is zero.

$+\circlearrowleft \quad \tau_{net} = (F_L \sin\theta) r - (F_a \sin\theta) R = 0$ when lever is in static equilibrium.

$\therefore \quad r F_L = R F_a$

(The force F_a is just enough to balance the weight mg. To pry up the stone $RF > rF$

Define mechanical advantage $MA = \frac{F_L}{F_a} = \frac{R}{r} > 1$

Clearly, $R > r \Rightarrow \frac{F_L}{F_a} > 1$ and $MA > 1$

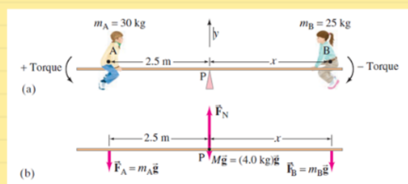
Need $MA \gg 1$ so we can carry heavy objects with small forces.

This means that we can lift a heavy object (F_L) by applying a small force (F_a)

So, in this case $MA = \frac{F_L}{F_a} = \frac{mg}{F_a} = \frac{R}{r} > 1$

9-2] Solving Static Problems

Balancing a seesaw. A board of mass serves as a seesaw for two children, as shown in Fig. a. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.



Balance the seasaw $\Rightarrow$ $\quad F_A x_A = F_B x_B$ for static equilibrium

① $\overset{+}{\uparrow} \; \Sigma F_y = F_N - Mg - m_A g - m_B g = 0$ (You can find F_N).

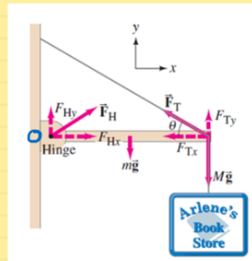
② $+\circlearrowleft \; \Sigma\tau = F_A(2.5) - F_B(x) = 0$.

$\therefore \; m_A g(2.5) = m_B g(x) \Rightarrow x = \frac{m_A}{m_B}(2.5) \Rightarrow$

$x = \frac{30}{25}(2.5) = 3\text{ m}$.

Example

Hinged beam and cable. A uniform beam, 2.20 m long with mass m= 25 kg is mounted by a small hinge on a wall as shown in the Figure. The beam is held in a horizontal position by a cable that makes an angle $\theta = 30^\circ$. The beam supports a sign of mass suspended from its end. Determine the components of the force that the (smooth) hinge exerts on the beam, and the tension in the supporting cable.



$M = 28\,kg$

I shall use H instead of F_H

T instead of F_T

Static equilibrium $\Rightarrow \Sigma F = 0$, $\Sigma \tau = 0$.

$\Sigma \tau = 0$

$$\overset{+}{\circlearrowleft} \quad \underbrace{(T\sin\theta)}_{F_{Ty}}(2.2) - mg(1.1) - Mg(2.2) = 0$$

$\therefore T \simeq 794\ N.$

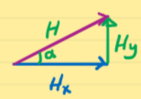
We still have two unknowns H_x and H_y (components of the hinge force. Need two equations:

$$\xrightarrow{+} \quad \Sigma F_x = 0 \Rightarrow H_x - T\cos 30 = 0$$

$\therefore H_x = \frac{\sqrt{3}}{2} T \sim 687.6\,N$ (+ $\Rightarrow$ direction of H_x is correct)

$$\overset{+}{\uparrow} \quad H_y + T\sin 30 - mg - Mg = 0$$

$\therefore H_y = 122.4\ N$ (+ $\Rightarrow$ drawn direction is correct)



$\tan\alpha = \left|\frac{H_y}{H_x}\right| \Rightarrow \alpha \sim 10.1^0$

$H = \sqrt{H_x^2 + H_y^2} \sim 698.4\ N.$