

# Chapter 6:Work and Energy

Lecture 2

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## Kinetic Energy

An object of mass  $m$  moving with velocity  $v$  has a form of energy called Kinetic Energy ( $K$ )

given by  $K = \frac{1}{2} m v^2$

clearly  $K \geq 0$ . If  $K=0 \Rightarrow$  object is at rest.

Work - kinetic energy theorem

Where does the work done on an object go?



FIGURE 6-7 A constant net force  $F_{\text{net}}$  accelerates a car from speed  $v_i$  to speed  $v_f$  over a displacement  $d$ . The net work done is  $W_{\text{net}} = F_{\text{net}}d$ .

In the figure, a net force acts on the car moves it a displacement  $\vec{d}$  in the direction of  $\vec{F}_{\text{net}}$  and hence accelerates it from an initial velocity  $\vec{v}_i$  to a final velocity  $\vec{v}_f$ . What is the work done on the car by the force  $\vec{F}_{\text{net}}$ ?

$$W = F_{\text{net}} d \cos(\alpha) = F_{\text{net}} d .$$

From the equations of motion, we have

$$v_f^2 - v_i^2 = 2ad$$

$$\times \frac{1}{2}m \Rightarrow \frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2 = \frac{1}{2}m a d \quad | \quad F_{\text{net}} = ma \\ = (ma)d \quad | \quad \text{Newton's 2nd law} \\ = F_{\text{net}} d$$

$$\therefore \underbrace{K_f - K_i}_{\substack{\Delta K \\ \text{Final kinetic energy} \\ \text{Initial kinetic energy}}} = W_{\text{net}} \quad \text{total (net) work done on the car.}$$

The total work done on an object equals the change in its kinetic energy.

$$W_{\text{net}} = \Delta K$$

$$W_{\text{net}} = \frac{1}{2}m(v_f^2 - v_i^2)$$

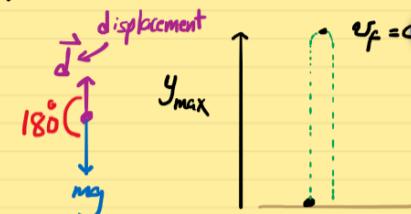
work-kinetic energy theorem

When  $W_{\text{net}} = 0 \Rightarrow \Delta K = 0 \Rightarrow K_f = K_i \Rightarrow$  speed does Not change.  
 $W_{\text{net}} > 0 \Rightarrow v_f > v_i \Rightarrow$  object accelerates,  $W_{\text{net}} < 0 \Rightarrow v_f < v_i \Rightarrow$  object decelerates.

Example: An object of mass  $m$  is projected vertically upwards from the earth's surface with an initial speed of 20 m/s.

① Find its maximum height.

Free Fall. Only force that acts on the object is its weight downwards. Displacement  $\vec{d}$  is upwards  $\Rightarrow \theta = 180^\circ$ .



$$W_{\text{net}} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$(mg)(y_{\text{max}}) \cos(180^\circ) = \frac{1}{2}m(0 - (20)^2)$$

$$\therefore -gy_{\text{max}} = -200 \Rightarrow y_{\text{max}} = 20 \text{ m.}$$

② Find the speed of the object when it is at a height of 15 m.

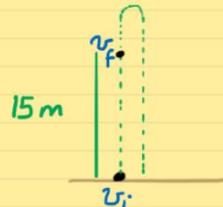
$$W_{\text{net}} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$(mg)(15) \cos 180^\circ = \frac{1}{2}m(v_f^2 - (20)^2)$$

$$-15 = \frac{1}{2}v_f^2 - 200$$

$$v_f^2 = 175$$

$$\therefore v_f = \sqrt{175} \text{ m/s.}$$



③ Find the speed of the object just before hitting the ground.

$$W_{\text{net}} = (mg)d \stackrel{\theta=90^\circ}{=} 0$$

$$\text{Return to ground} \Rightarrow d = 0 \Rightarrow W_{\text{net}} = 0$$

$$0 = \frac{1}{2}m(v_f^2 - v_i^2) \Rightarrow |v_f| = |v_i| = 20 \text{ m/s (speeds)}$$

[for velocities  $v_f = \pm v_i$ ]

If  $\uparrow v_f = -v_i = -20 \text{ m/s}$ , if  $\downarrow v_f = +20 \text{ m/s}$ .

**CONCEPTUAL EXAMPLE 6-5** Work to stop a car. A car traveling 60 km/h can brake to a stop in a distance  $d$  of 20 m (Fig. 6-10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6-10b)? Assume the maximum braking force is approximately independent of speed.

Stopping  
distance  
 $\equiv d$

$$v_i = 2v_i$$

what is  
 $d'$ ?  
now  
stopping  
distance.

**RESPONSE** Again we model the car as if it were a particle. Because the net stopping force  $F$  is approximately constant, the work needed to stop the car,  $Fd$ , is proportional to the distance traveled. We apply the work-energy principle, noting that  $\vec{F}$  and  $\vec{d}$  are in opposite directions and that the final speed of the car is zero:

$$\begin{aligned} W_{\text{net}} &= Fd \cos 180^\circ = -Fd. \\ \text{Then } -Fd &= \Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \\ &= 0 - \frac{1}{2}mv_1^2. \end{aligned}$$

Thus, since the force and mass are constant, we see that the stopping distance,  $d$ , increases with the square of the speed:

$$d \propto v^2. \quad \text{Stopping distance } d \propto v_i^2$$

If the car's initial speed is doubled, the stopping distance is  $(2)^2 = 4$  times as great, or 80 m.

$$d' = \frac{m}{2F} (2v_i)^2 = 4 \left( \frac{m}{2F} v_i^2 \right) = 4d$$

$$\text{at } v_i = 60 \text{ km/h}, d = 20 \text{ m} \Rightarrow \text{at } 120 \text{ km/h}, d' = 4d = 4(20) = 80 \text{ m.}$$

$$Fd = \frac{1}{2}mv_i^2$$

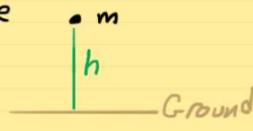
$$d = \frac{m}{2F} v_i^2$$

$$d \propto v_i^2$$

## 6-4] Potential Energy

### Gravitational Potential Energy

Object of mass  $m$  is at a height  $h$  above the surface of the ground.



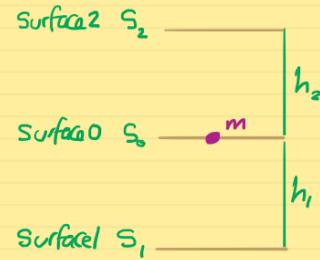
If you release this object it falls towards the ground. This shows that the object possessed energy when it was at height  $h$  above the ground. This energy is called the gravitational potential energy which is the energy possessed by the object due to its position above the ground.

gravitational potential energy  $\rightarrow U = mgh$



Gravitational potential energy is defined with respect to a surface.

$U_2 = -mgh_2$  potential energy of mass m relative to  $S_2$ . Negative  
 $U_2$  means work must be done to raise m to  $S_2$ .

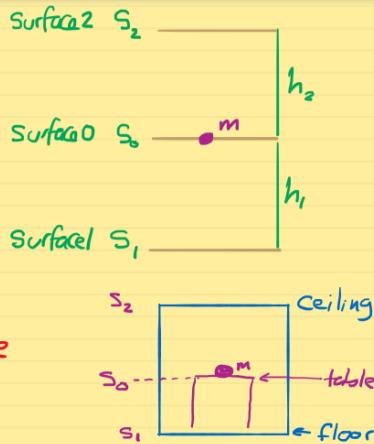


$U_1 = mgh_1$  potential energy of mass

$U_2 = -mgh_2$  potential energy of mass m relative to  $S_2$ . Negative  
 $U_2$  means work must be done to raise m to  $S_2$ .

$U_1 = mgh_1$  potential energy of mass m relative to  $S_1$ . Positive value means if you release m it falls towards  $S_1$ .

$mgh = mg(d) = 0$   
 $U_0 = 0$  potential energy of mass m with respect to surface  $S_0$ . Note m is on the surface  $S_0 \Rightarrow h = 0$ .



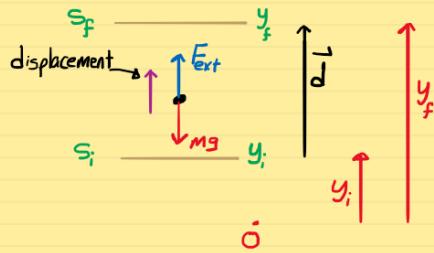
Unlike kinetic energy  $K$ , the potential energy can be positive, negative or zero.

Find the work done by the weight ( $mg$ ) while an object is moved from surface  $s_i \rightarrow s_f$ .

$$W_{mg} = (mg)(d) \cos 180^\circ \\ = mg(y_f - y_i)(-1)$$

$$W_{mg} = - (mgy_f - mgy_i)$$

$$W_{mg} = - (U_f - U_i) = - \Delta U$$



If the mass is moved up at constant velocity ( $a=0$ )  $\Rightarrow F_{ext} = mg$ .

Work done by the external force is

$$W_{ext} = (F_{ext})(d) \cos(0) = mgd = mg(y_f - y_i)$$

$$W_{ext} = U_f - U_i = \Delta U$$

$$\therefore W_{ext} = \Delta U, W_{mg} = -\Delta U$$

$$\vec{F}_{ext} \uparrow d \quad \theta = 0$$

$W_{net} = W_{mg} + W_{ext} = 0 \Rightarrow$  object moves at constant speed upwards.

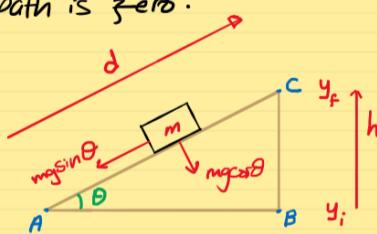
## Conservative and nonconservative forces.

Properties of conservative forces  
*(The work done by)*

(i) The work of a conservative force does  
NOT depend on the path.

(ii) The work of a conservative force  
round a closed path is zero.

Find the work done  
by the force of  
gravity ( $mg$ ) when  
the object of mass  
 $m$  moves from



round a closed path is zero.

Find the work done  
by the force of  
gravity ( $mg$ ) when  
the object of mass  
 $m$  moves from

point A to C through two different paths:

①  $A \rightarrow C$  directly.

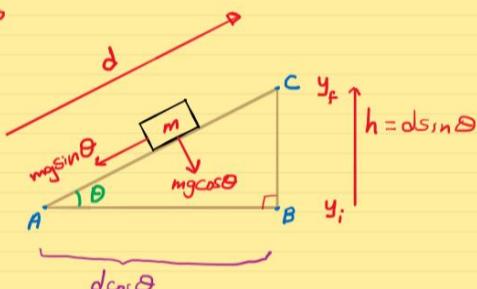
①  $A \rightarrow C$  directly.

$$W_{mg}(A \rightarrow C) = (mg \sin \theta)(d) \cos 180^\circ$$

$$W_{mg}(A \rightarrow C) = -mgd \sin \theta$$

$$\text{But } h = d \sin \theta \Rightarrow$$

$$W_{mg}(A \rightarrow C) = -mgh.$$



② Find the work done by the force of gravity in  
moving the mass  $m$  from  $A \rightarrow B \rightarrow C$

$$W_{mg}(A \rightarrow B \rightarrow C) = W_{mg}(A \rightarrow B) + W_{mg}(B \rightarrow C)$$

$$W_{mg}(A \rightarrow B) = (mg)(d_i) \cos 90^\circ$$

$$\therefore W_{mg}(A \rightarrow B) = 0$$

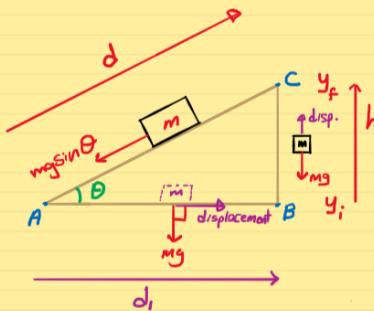
$$W_{mg}(B \rightarrow C) = (mg)(h) \cos 180^\circ \\ = -mgh$$

$$\therefore W_{mg}(A \rightarrow B \rightarrow C) = 0 + (-mgh)$$

$W_{mg}(A \rightarrow B \rightarrow C) = -mgh$  as before.

$$\therefore W_{mg}(A \xrightarrow{\text{directly}} C) = W_{mg}(A \rightarrow B \rightarrow C) = -mgh$$

$\therefore$  Work of gravity does NOT depend on the path.



③ Find the work done by gravity when the object moves from  $A \rightarrow B \rightarrow C \rightarrow A$  (closed path).

$$W_{mg}(A \rightarrow B \rightarrow C \rightarrow A)$$

$$= W_{mg}(A \rightarrow B \rightarrow C) + W_{mg}(C \rightarrow A)$$

$$= -mgh + mgh = 0$$

Note:  $mgsin\theta$   
is parallel to  $d$   
 $\Rightarrow$  angle between them = 0

$$\therefore W_{mg}(A \rightarrow B \rightarrow C \rightarrow A) = 0$$

$\therefore$  Work of gravity round a closed path = 0.

$$W(C \rightarrow A) = (mg \sin \theta)(d) \cos(0) \\ = mgd \sin \theta \\ = mgh$$

