

Chapter 6: Work and Energy

Lecture 4
(First Sem 20/21)
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6-9] Energy conservation with dissipative forces

We live in a world where nonconservative forces like friction, tension, applied forces do work. The work done by such nonconservative forces (W_{nc}) is taken into account using

$$\Delta K + \Delta U = W_{nc}$$

$$(K_f - K_i) + (U_f - U_i) = W_{nc}$$

$$(K_f + U_f) - (K_i + U_i) = W_{nc}$$

$$E_f - E_i = W_{nc}$$

$$(K_f - K_i) + (U_f - U_i) = W_{nc}$$

$$(K_f + U_f) - (K_i + U_i) = W_{nc}$$

$$E_f - E_i = W_{nc}$$

$\therefore \Delta E = W_{nc}$

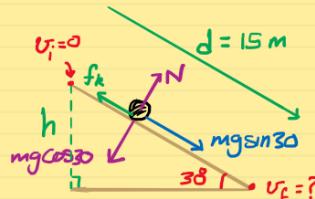
↑ ↗
change in total work done by nonconservative
mechanical energy forces

\therefore change in total mechanical energy equals the work by nonconservative forces.

Example: Starting from rest a skier slides down a 30° inclined plane a distance of 15 m. If the coefficient of kinetic friction is 0.1 find his speed at the bottom of the slide.

Note N and $mg\cos 30$ do No work.

$$\Delta K + \Delta U = W_{nc}$$



$$\frac{1}{2}m(v_f^2 - v_i^2) - mg(h\sin 30) = W_{nc}$$

h: vertical distance descended by the skier

$$\frac{1}{2}m(v_f^2 - v_i^2) - mg(15 \times \frac{1}{2}) = f_k d \cos 180^\circ$$

$$\frac{1}{2}m(v_f^2 - v_i^2) - mg(15 \times \frac{1}{2}) = (\mu_k mg \cos 30)(15)(-1)$$

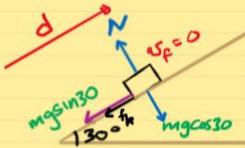
$$\therefore v_f^2 = \frac{15g}{2} - (0.1)(g)(\frac{\sqrt{3}}{2})(15) \Rightarrow v_f = 7.8 \text{ m/s.}$$

Example A box of mass m is given an initial speed of 12 m/s up a 30° inclined plane. If the coefficient of kinetic friction between the box and the plane is $\mu_k = 0.15$ find the maximum distance the box moves up the inclined plane.

up the inclined plane.

N and $mg\cos 30$ do No work.

$$\Delta K + \Delta U = W_{nc}$$



$$\frac{1}{2}m(v_f^2 - v_i^2) + mg(h\sin 30) = f_k d \cos 180^\circ$$

$$\frac{1}{2}m(0 - (12)^2) + mg(\frac{d}{2}) = (\mu_k mg \cos 30)(d)(-1)$$

$$-72 = -d(\mu_k g \cos 30 + \frac{1}{2}g) \Rightarrow d = 11.7 \text{ m.}$$

Example

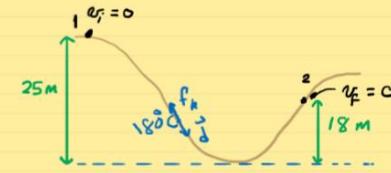
A roller-coaster car starts from rest at point ① and slides down the roller-coaster. The car moves a distance of 60 m on the track before coming to rest at point ②. Find the magnitude of the force of kinetic friction f_k (Assume f_k to be constant)

$$\Delta K + \Delta U = W_{nc}$$

$$\frac{1}{2}m(v_f^2 - v_i^2) - mg(7) = f_k d \cos 180^\circ$$

$$\frac{1}{2}m(0 - 0) - 7mg = -f_k(60)$$

$$\therefore f_k = \frac{7mg}{60} = \frac{(7)(150)(10)}{60} = 175 \text{ newtons.}$$

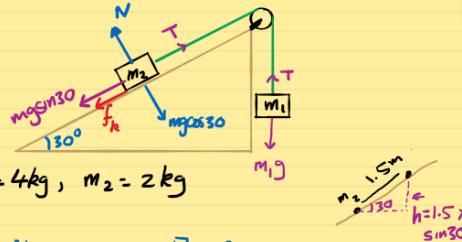


Example

In the figure μ_k between m_2 and the inclined plane is 0.2. Find the speed of m_1 after it has fallen a distance of 1.5 m. Assume system started from rest. Also $m_1 = 4\text{kg}$, $m_2 = 2\text{kg}$

$$\Delta K + \Delta U = W_{nc}$$

$$[\frac{1}{2}m_1(v_f^2 - v_i^2) - mg(1.5)] + [\frac{1}{2}m_2(v_f^2 - v_i^2) + mg(1.5 \sin 30^\circ)] = f_k(1.5) \cos 180^\circ$$



Note # since m_1 and m_2 are connected by an inextensible string \Rightarrow they have the same speed.

when determining ΔU we only need the vertical distance ascended (for m_2) or descended for (m_1)

$$1.5 \sin 30^\circ$$

$$1.5 \text{ m}$$

$$\frac{1}{2}(m_1+m_2)v_f^2 + 1.5g[m_2 \sin 30 - m_1] = \mu_k m_2 g \cos 30 \times (-1)$$

$$34v_f^2 + [-34.02] = -3.46$$

$$\therefore v_f \approx 3.2 \text{ m/s.}$$

6-10] Power (P)

Power: Work done per unit time (rate of doing work)

$$\text{Average Power } \bar{P} = \frac{\text{Work done}}{\text{time taken}}$$

$$\text{unit of power [P]} = \frac{\text{J}}{\text{s}} \equiv \text{Watt (W)}$$

$$\therefore 1 \text{ W} = 1 \text{ J/s.}$$

Another unit is the horsepower (hp)

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$$1 \text{ hp} = 746 \text{ Watt.}$$

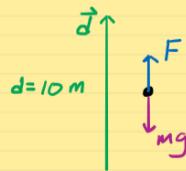
$$\begin{array}{l} 10 \text{ W} \\ 10 \text{ J/s} \end{array}$$

Example

A 60-kg firefighter climbs a 10 m vertical rope in 10 seconds at constant speed. Find his average power output.

The firefighter exerts a force F as he climbs the rope.

$$\Rightarrow \bar{P} = \frac{W_F}{t} = \frac{Fd \cos(0)}{t} = \frac{Fd}{t}$$



How can we find F ? He moves upwards at constant speed ($a_y = 0$).

Using Newton's second law

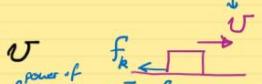
$$\begin{aligned} \uparrow \quad \sum F_y &= ma_y \Rightarrow F - mg = m(0) \\ \therefore F &= mg \end{aligned}$$

$$\bar{P} = \frac{W}{t}$$

$W = \bar{P} t$

$$\therefore \bar{P} = \frac{mgd}{t} = \frac{(60)(10)(10)}{10} = 600 \text{ W}.$$

NOTE: using $\bar{P} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$



$$\bar{P} = Fv \cos\theta$$

ONLY when speed is constant.

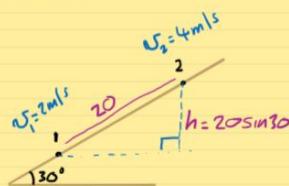
angle between F and v .

$$\bar{P} = f_k v \cos 60^\circ$$

$$\bar{P} = mg\left(\frac{10}{10}\right) = mg(1) = 60 \times 10 \times 1 = 600 \text{ J}$$

As before.

Example (mass = 50 kg)
A runner is going up a 30° inclined plane. At point 1 his speed is 2 m/s while at point 2 his speed is 4 m/s. The distance between points 1 and 2 is 20 m. He takes 10 s to move from point 1 \rightarrow point 2.



If his mass is 50 kg find his average power output.

$$\bar{P} = \frac{W}{t}$$

How to find W ?

Remember, the force of the runner is a nonconservative force \Rightarrow

$$W = W_{nc} = \Delta K + \Delta U$$

$$= \frac{1}{2}m(v_f^2 - v_i^2) + mg(20 \sin 30)$$

when running at constant speed this term = 0 as in the above example of the fire fighter.

$$\therefore W = W_{nc} = \frac{1}{2}(50)[16 - 4] + 50(10)(20 \sin 30)$$

vertical distance ascended

$$= 5300 \text{ J}$$

$$\Rightarrow \bar{P} = \frac{5300}{10} = 530 \text{ Watt}$$

i.e. 530 J/s.



for fire fighter



$$W_F = W_{nc} = \Delta K + \Delta U$$
$$= 0 + mg(10)$$

$$= 60 \times 10 \times 10 = 6000 \text{ J}$$

$$\bar{P} = \frac{W_F}{t} = \frac{6000}{10} = 600 \text{ Watt as before.}$$

$$\int_{10\text{m}} \uparrow \text{Const. } U$$