Chapter 6:Work and Energy

Lecture 1
(First Sem 20/21)
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6-1] Work done by a constant force.

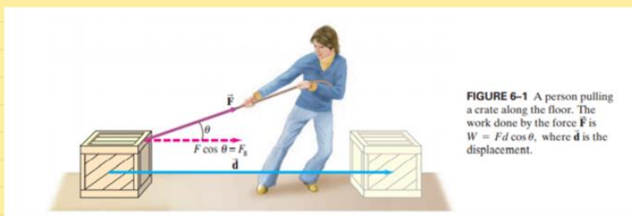


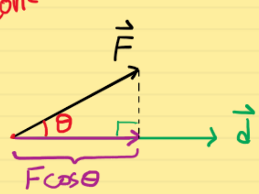
FIGURE 6–1 A person pulling a crate along the floor. The work done by the force **F** is $W = Fd\cos\theta$, where **d** is the displacement.

To move the box a displacement $\vec{d}$ to the right as shown in the figure, work must be done

How to define what is meant by work?

Work done = magnitude of the displacement times the component of the force parallel to the displacement.

$$\text{Work done} \rightarrow W = (F\cos\theta)\,d = F\,d\cos\theta$$



θ: smaller angle between $\vec{F}$ and $\vec{d}$ when they originate from the same point.

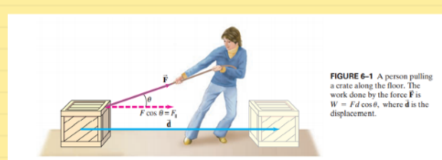
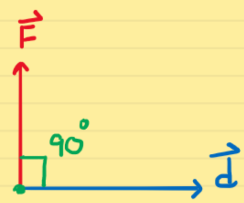


FIGURE 6–1 A person pulling a crate along the floor. The work done by the force **F** is $W = Fd\cos\theta$, where **d** is the displacement.

NOTE: Work is a scalar quantity.

When does the work done by a Force $\vec{F}$ has a value of zero?

$$W = Fd\cos\theta$$

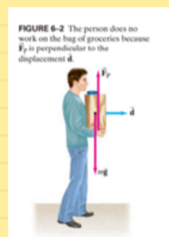


$W = 0$ when:

$\theta = 90°$

$d = 0$.

In the figure the work done by the man is zero, because his force is perpendicular to the displacement ($\theta = 90°$)



$W = Fd\cos\theta$

Unit of Work:

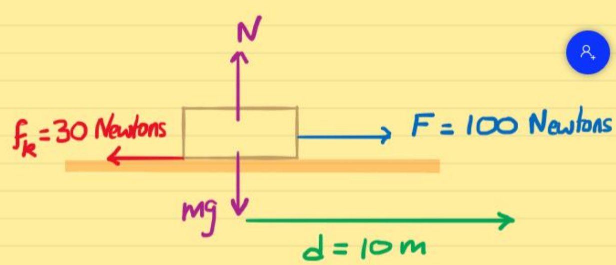
unit of work $\rightarrow$ $[W] = N \cdot m \equiv$ Joule

$1\,J = 1\,N \cdot 1\,m$

When a force of 1N acts on an object and moves it a distance of 1m along the direction of the force $\Rightarrow$ the work done is 1 Joule.

Example:

In the figure the box moves a distance of 10m to the right.



(i) Find the work done by each force.

$W_N = N\,d\cos(90°) = 0$

$W_{mg} = mg\,d\cos(90°) = 0$

$W_F = Fd\cos(0) = (100)(10)(1) = 1000\,J$. (NOTE $W > 0$) F moves the object

$W_{mg} = mg\,d\cos(90^\circ) = 0$

$W_F = Fd\cos(0) = (100)(10)(1) = 1000\text{ J}$. F moves the object (NOTE $W>0$)

$W_{f_k} = f_k\,d\cos 180^\circ = (30)(10)(-1) = -300\text{ J}$. ($W<0$) f_k impedes the motion of the box.

Note: the work of friction is negative. BUT work is a scalar and NOT a vector. The sign here does NOT mean direction.

(ii) Find the net (total) work done on the box.

$$W_{net} = W_N + W_{mg} + W_F + W_{f_k}$$
$$= 0 + 0 + 1000 + (-300) = 700\text{ J}.$$

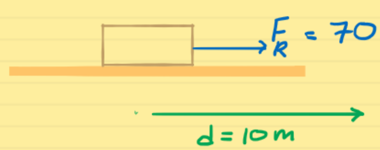
Alternatively:

$\xrightarrow{+}$ $F_R = F - f_k = 100 - 30 = 70$ Newtons in the positive x-direction.

$+\uparrow$ NOTE: $N - mg = 0$.

$$W_{net} = \underbrace{F_R}_{\text{net (resultant) force}}\,d\cos\theta$$
$$= (70)(10)\cos(0)$$
$$= 700\text{ J}.$$

As before.



A resultant force acts on the box in the positive x-direction $\Rightarrow$ the box accelerates along the positive x-direction. Remember the net work done on the object is positive.

What does the sign of the work done on an object mean?

$W_{net} > 0$ (i.e positive) $\Rightarrow$ object accelerates

$W_{net} < 0$ (i.e negative) $\Rightarrow$ object decelerates

$W_{net} = 0$ $\Rightarrow$ object moves at constant speed.

$W_{net} > 0$ (i.e positive) $\Rightarrow$ object accelerates

$W_{net} < 0$ (i.e negative) $\Rightarrow$ object decelerates

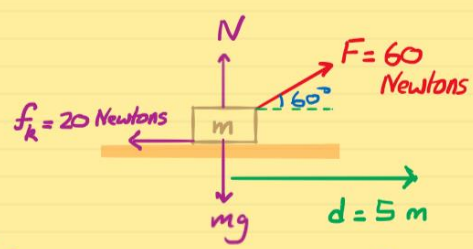
$W_{net} = 0$ $\Rightarrow$ object moves at constant speed.

$W_F = 1000\,J > 0$ $\Rightarrow$ F accelerates the box.

$W_{f_k} = -300\,J < 0$ $\Rightarrow$ f_k impedes the motion of the box and decelerates the box.

$W_{net} = 700\,J$

Example: Find the net work done on the box.



$W_N = W_{mg} = 0$

$W_F = (60)(5)\cos 60 = 150\,J$.

$W_{f_k} = (20)(5)\cos 180 = -100\,J$

$\Rightarrow W_{net} = 150 - 100 = 50\,J$

EXAMPLE 6–2 **Work on a backpack.** (*a*) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0$ m, as shown in Fig. 6–4a. Determine also (*b*) the work done by gravity on the backpack, and (*c*) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).

APPROACH We explicitly follow the steps of the Problem Solving Strategy above.

SOLUTION

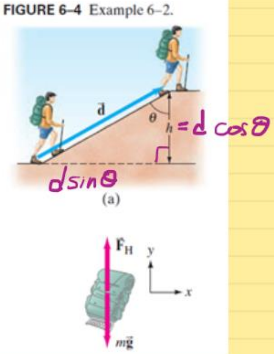
1. **Draw a free-body diagram.** The forces on the backpack are shown in Fig. 6–4b: the force of gravity, $m\vec{g}$, acting downward; and $\vec{F}_H$, the force the hiker must exert upward to support the backpack. The acceleration is zero, so horizontal forces on the backpack are negligible.
2. **Choose a coordinate system.** We are interested in the vertical motion of the backpack, so we choose the y coordinate as positive vertically upward.
3. **Apply Newton's laws.** Newton's second law applied in the vertical direction to the backpack gives (with $a_y = 0$)

$$\Sigma F_y = ma_y$$
$$F_H - mg = 0.$$

So,

$$F_H = mg = (15.0\,\text{kg})(9.80\,\text{m/s}^2) = 147\,\text{N}.$$

FIGURE 6–4 Example 6–2.



F_H ↑ ↑h $\Rightarrow \theta = 0$

↓ ↑h mg $\theta = 180°$

$W_{F_H} = (F_H)(h)\cos(0) = 147 \times 10 = 1470\,J$

$W_{mg} = (mg)(h)\cos 180° = 147 \times 10(-1) = -1470\,J$

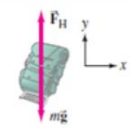
$\Big\} \; W_{net} = 0 \Rightarrow$ constant speed.

3. **Apply Newton's laws.** Newton's second law applied in the vertical direction to the backpack gives (with $a_y = 0$)

$$\Sigma F_y = ma_y$$
$$F_H - mg = 0.$$

So,

$$F_H = mg = (15.0\,\text{kg})(9.80\,\text{m/s}^2) = 147\,\text{N}.$$



(b)

4. **Work done by a specific force.** (*a*) To calculate the work done by the hiker on the backpack, we use Eq. 6–1, where θ is shown in Fig. 6–4c,

$$W_H = F_H(d\cos\theta),$$

and we note from Fig. 6–4a that $d\cos\theta = h$. So the work done by the hiker is

$$W_H = F_H(\underline{d\cos\theta}) = F_H\underline{h} = mgh = (147\,\text{N})(10.0\,\text{m}) = 1470\,\text{J}.$$

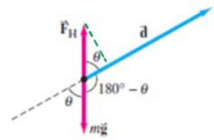
The work done depends only on the elevation change and not on the angle of the hill, θ. The hiker would do the same work to lift the pack vertically by height h.

(*b*) The work done by gravity on the backpack is (from Eq. 6–1 and Fig. 6–4c)

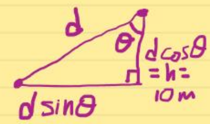
$$W_G = mg\,d\cos(180^\circ - \theta).$$

Since $\cos(180^\circ - \theta) = -\cos\theta$ (Appendix A–7), we have

$$W_G = mg(-d\cos\theta)$$
$$= -mgh$$
$$= -(15.0\,\text{kg})(9.80\,\text{m/s}^2)(10.0\,\text{m}) = -1470\,\text{J}.$$



(c)



NOTE The work done by gravity (which is negative here) does not depend on the angle of the incline, only on the vertical height h of the hill.

5. **Net work done.** (*c*) The *net* work done on the backpack is $W_{net} = 0$, because the net force on the backpack is zero (it is assumed not to accelerate significantly). We can also get the net work done by adding the work done by each force:

$$W_{net} = W_G + W_H = -1470\,\text{J} + 1470\,\text{J} = 0.$$

PROBLEM SOLVING
Work done by gravity depends on height of hill (not on angle)

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moon orbits the earth at constant speed.

Parallel
$\vec{d} \parallel \vec{v}$

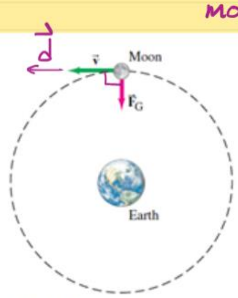


FIGURE 6–5 Example 6–3.

CONCEPTUAL EXAMPLE 6–3 **Does the Earth do work on the Moon?** The Moon revolves around the Earth in a nearly circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work on the Moon?

RESPONSE The gravitational force $\vec{F}_G$ exerted by the Earth on the Moon (Fig. 6–5) acts toward the Earth and provides its centripetal acceleration, inward along the radius of the Moon's orbit. The Moon's displacement at any moment is tangent to the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force $\vec{F}_G$ and the instantaneous displacement of the Moon is 90°, and the work done by gravity is therefore zero ($\cos 90^\circ = 0$). This is why the Moon, as well as artificial satellites, can stay in orbit without expenditure of fuel: no work needs to be done against the force of gravity.

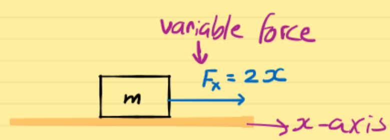
F_G towards the center of the earth.

$\vec{d}$ is always perpendicular to $\vec{F}_G \Rightarrow \theta = 90^\circ$

$W_{net} = 0$

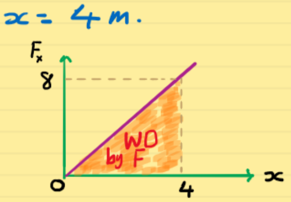
6-2] Work done by a variable force.

A box moves along the x-axis under the effect of a variable force $F_x = 2x$.



Find the work done by F_x when the box moves from $x = 0\,\text{m} \longrightarrow x = 4\,\text{m}$.

W = Area under the curve



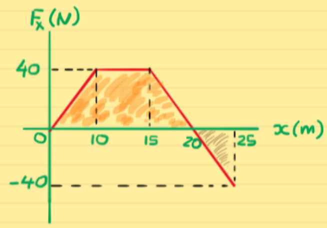
$W = \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(4)(8) = 16\,\text{J}$.

$\text{m} \cdot \text{N} \equiv \text{J}$

For $F_x - x$ graph area under the curve gives the work done.

Example: An object moves along the x-axis under a variable force F_x as shown in the Figure.



Find the work done on the box by F_x over the following intervals:

i) $x = 0 \rightarrow 20$ m

$W_1 = \frac{1}{2}(5+20)(40) = 500\,J$

ii) $x = 20 \rightarrow 25$ m

$W_2 = \frac{1}{2}(5)(-40) = -100\,J$

iii) Find the net work done on the box by F_x.

$W_{net} = W_1 + W_2 = 500 - 100 = 400\,J$.