

Chapter 30: Nuclear Physics and Radioactivity

Lecture 2

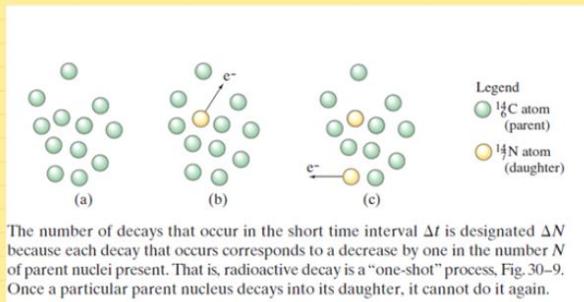
(First Sem 20/21)

The University of Jordan/Physics Department

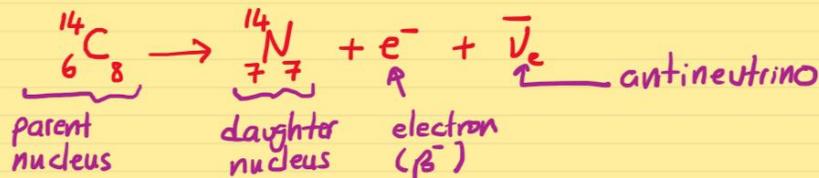
Prof. Mahmoud Jaghoub

أ.د. محمود الجاغوب

30-8] Half-Life and Rate of Decay.



${}^{14}_6\text{C}$ is an unstable nucleus \Rightarrow it decays by emitting β^- particles (e^-) and turns into ${}^{14}_7\text{N}$ which is stable.



As time passes, the number of ${}^{14}_6\text{C}$ radioactive nuclei decreases, while the number of daughter stable ${}^{14}_7\text{N}$ nuclei increases.

How does the number of ${}^{14}_6\text{C}$ nuclei change with time?

This is given by the radioactive decay law.

$$N = N_0 e^{-\lambda t}$$

\log_{10} , $\log_e \leftarrow \ln$
↑ natural logarithm.

N : remaining number of radioactive nuclei at time t .

$$\log_{10}^{100} = 2 \Rightarrow 10^2 = 100$$

N_0 : initial number of radioactive nuclei at time $t_0 = 0$.

λ : decay constant. Has units of $\frac{1}{\text{time}}$
for example s^{-1} , min^{-1} , etc.

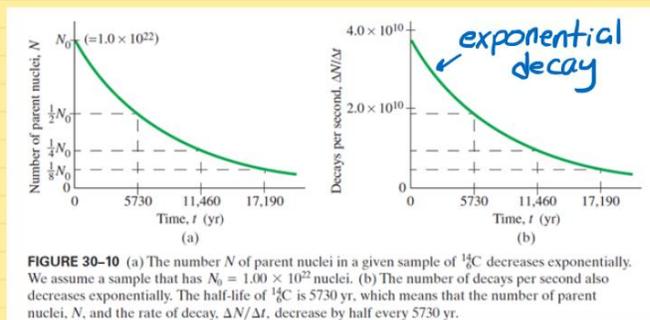
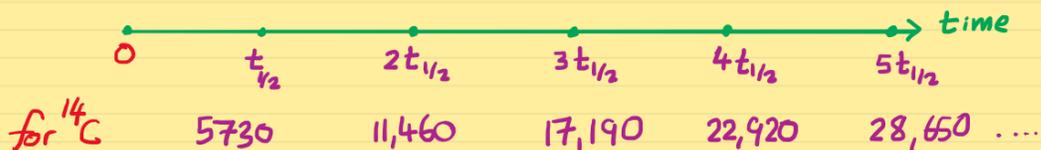


FIGURE 30-10 (a) The number N of parent nuclei in a given sample of ^{14}C decreases exponentially. We assume a sample that has $N_0 = 1.00 \times 10^{22}$ nuclei. (b) The number of decays per second also decreases exponentially. The half-life of ^{14}C is 5730 yr, which means that the number of parent nuclei, N , and the rate of decay, $\Delta N/\Delta t$, decrease by half every 5730 yr.

half-life: time needed for the number of radioactive nuclei to become half the original number.

$$N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8} \xrightarrow{t_{1/2}} \frac{N_0}{16} \xrightarrow{t_{1/2}} \frac{N_0}{32} \dots$$



Calculate the half-life ($t_{1/2}$) in terms of the decay constant.

remaining number $\rightarrow N = N_0 e^{-\lambda t}$, after $t_{1/2}$ we have $N \rightarrow \frac{N_0}{2}$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln e^x = x$$

$$\ln \frac{1}{2} = \ln e^{-\lambda t_{1/2}} = -\lambda t_{1/2}$$

↑
natural logarithm

$$-\ln\left(\frac{1}{2}\right) = \ln\left(\frac{1}{2}\right)^{-1} = \ln 2$$

$$\therefore t_{1/2} = -\frac{\ln\left(\frac{1}{2}\right)}{\lambda} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Each radioactive nuclide has its own λ .

large $\lambda \Rightarrow$ small $t_{1/2} \Rightarrow$ fast decay.

small $\lambda \Rightarrow$ large $t_{1/2} \Rightarrow$ slow decay.

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

For ^{238}U : $t_{1/2} \approx 4.5 \times 10^9 \text{ yr} \Rightarrow \lambda = \frac{\ln 2 \times 10^{-9}}{4.5} \text{ yr}^{-1}$

very small $\lambda \Rightarrow$ very large $t_{1/2} \Rightarrow$ slow decay.

For Iodine-131 (^{131}I) (used to treat thyroid gland cancer and also treatment of an overactive thyroid gland).

$$t_{1/2} \approx 8 \text{ days}$$

$$t_{1/2} \approx 8 \text{ days} \Rightarrow \lambda = \frac{\ln 2}{8} \approx 0.087 \text{ day}^{-1}$$

large λ (compared to ^{238}U) \Rightarrow small $t_{1/2} \Rightarrow$ fast decay.

Activity (A) : number of decays per second .
[rate of decay]

$$N = N_0 e^{-\lambda t}$$

$$A = -\frac{dN}{dt} = -[N_0(-\lambda) e^{-\lambda t}]$$

$$\therefore A = \lambda N_0 e^{-\lambda t}$$

$$A = \lambda N$$

$$A = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

activity at
time t .

initial activity

[The book uses R instead of A]

Mean life (τ): average life time of all the
radioactive nuclei of a given radioactive
element .

(τ)

$$\tau = \frac{1}{\lambda} = \frac{1}{\left(\frac{\ln 2}{t_{1/2}}\right)} = \frac{t_{1/2}}{\ln 2}$$

30-9] Calculations Involving activity
Decay Rate
and Half-life .

Example 30-9] Sample activity. The isotope ^{14}C has a half-life of 5730 yr. If a sample contains 1.0×10^{22} carbon-14 nuclei, what is the activity of the sample?

$A = \lambda N \Rightarrow$ need to calculate λ .

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{\underbrace{5730}_{\text{yr}} \times \underbrace{365 \times 24 \times 60 \times 60}_{\frac{\text{s}}{\text{yr}}}} = 3.84 \times 10^{-12} \text{ s}^{-1}$$

$$\therefore A = (3.84 \times 10^{-12} \text{ s}^{-1})(1.0 \times 10^{22})$$

$$A = 3.84 \times 10^{10} \text{ decays/s.} \quad \text{unit of activity}$$

$$\text{Can say } A = 3.84 \times 10^{10} \text{ s}^{-1}.$$

The SI unit of activity is the Becquerel (Bq)

$$1 \text{ Bq} = 1 \text{ decays/s or simply } \text{s}^{-1}$$

Another more common unit of activity is the curie (Ci)

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/s}$$

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq.}$$

Example 30-11] A laboratory has 1.49 μg of pure $^{13}_7\text{N}$, which has a half-life of 10.0 min.

a) How many nuclei are present initially?

molar mass

Remember: mass of one mole of a substance contains Avogadro's number of the particles or molecules of that substance. (Avogadro's number $N_A = 6.02 \times 10^{23}$)

$$\begin{matrix} \text{micro} \\ \mu \equiv 10^{-6} \end{matrix}$$

$$\begin{matrix} \text{m} & \equiv & 10^{-3} \\ \mu & \equiv & 10^{-6} \\ \text{n} & \equiv & 10^{-9} \\ \text{p} & \equiv & 10^{-12} \end{matrix}$$

Molar mass equals the mass number in units of grams.

Nuclide	molar mass (grams/mole)
¹² C	12
¹³ N	13
¹³¹ I	131

13 grams of ¹³N → 1 mole

1.49×10^{-6} grams of ¹³N → x mole

$$\begin{aligned} x &\rightarrow y \\ z &\rightarrow a \end{aligned}$$

$$ax = zy$$

$$a = \frac{zy}{x}$$

∴ number of moles of ¹³N is

$$x = \frac{1.49 \times 10^{-6} \text{ grams} \times 1 \text{ mole}}{13 \text{ grams}} \approx 1.146 \times 10^{-7} \text{ mole}$$

∴ Number of nuclei of ¹⁴N is $x \times N_A \approx 6.9 \times 10^{16}$ nuclei

b) What is the initial activity (initial decay rate)?

$$\begin{aligned} A &= A_0 e^{-\lambda t} \\ &= \lambda N_0 e^{-\lambda t} \end{aligned}$$

$$\therefore A_0 = \lambda N_0$$

← 6.9×10^{16}
↑ need to calculate λ

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{10 \times 60} \approx 0.001155 \text{ s}^{-1}$$

$$\therefore A_0 \approx 7.97 \times 10^{13} \text{ Bq} \quad \leftarrow 1 \text{ decay/s} \quad m = 1.49 \times 10^{-6} \text{ grams}$$

Find out the activity of 1.49 μg of ^{238}U .

c) What is the activity after 1h?

$$A = A_0 e^{-\lambda t} = 7.97 \times 10^{13} e^{-\lambda t}$$

$$\lambda t = \frac{\ln 2}{t_{1/2}} \times t = \frac{\ln 2}{10 \text{ min}} \times 60 \text{ min}$$

$$\lambda t = 6 \ln 2$$

$$\therefore A = 7.97 \times 10^{13} \times e^{-6 \ln 2}$$

$$\approx 1.25 \times 10^{12} \text{ Bq} = 1.25 \times 10^{12} \cancel{\text{Bq}} \times \frac{1 \text{ Ci}}{3.7 \times 10^{10} \cancel{\text{Bq}}}$$

$$\therefore A \approx 33.8 \text{ Ci}$$

Alternatively:

$$1 \text{ h} = 60 \text{ min} = 6 \times 10 = 6 t_{1/2}$$

$$N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8} \xrightarrow{t_{1/2}} \frac{N_0}{16} \xrightarrow{t_{1/2}} \frac{N_0}{32} \xrightarrow{t_{1/2}} \frac{N_0}{64}$$

So, the activity will decrease to:

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$\text{i.e. } A \rightarrow A_0 \times \frac{1}{64} = \frac{7.97 \times 10^{13}}{64} \approx 1.25 \times 10^{12} \text{ Bq} \text{ as before.}$$

d) After approximately how long will the activity drop to less than 1 decay/s?

$$A = A_0 e^{-\lambda t}$$
$$1 = 7.97 \times 10^{13} e^{-\frac{\ln 2}{600} \times t}$$

$$\ln\left(\frac{1}{7.97 \times 10^{13}}\right) = -\frac{\ln 2}{600} t$$

$$\therefore t \approx 2.7901 \times 10^4 \text{ s}$$

$$\approx 7.75 \text{ hr}$$

$$\therefore \text{after approximately } 2.7901 \times 10^4 \text{ s}$$