

**Chapter 3:Kinematics in Two Dimensions, Vectors**

Lecture 2  
(First Sem 20/21)

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Microsoft Whiteboard

3-3] Subtraction of vectors .

First need to introduce two important points :

1) Equality of two vectors.

If  $\vec{A} = \vec{B}$   $\Rightarrow$

(i)  $|\vec{A}| = |\vec{B}|$  i.e they have equal magnitudes

(ii)  $\vec{A}$  is parallel to  $\vec{B}$  i.e they are in the same direction.  
NOTE that both conditions must be satisfied.



Since vectors  $\vec{A}$  and  $\vec{B}$  have the same length  $\Rightarrow |\vec{A}| = |\vec{B}|$

## 2) Negative of a vector

If  $\vec{A} = -\vec{B} \Rightarrow$

(i)  $|\vec{A}| = |\vec{B}|$

(ii)  $\vec{A}$  is antiparallel to  $\vec{B}$  i.e they are in opposite directions.



Now can find  $\vec{A} - \vec{B}$  as follows: we turn subtraction into addition:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

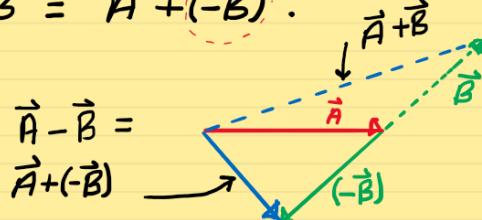
so we add  $\vec{A}$  to vector  $(-\vec{B})$ .

Suppose



Find  $\vec{A} - \vec{B}$ . add  $-\vec{B}$  to vector  $\vec{A}$ .

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

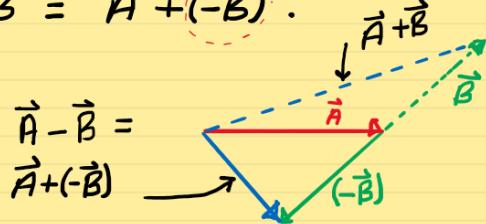


Remember:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \text{ commutative}$$

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

Find  $\vec{A} - \vec{B}$ .  
 $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ . add  $-\vec{B}$  to vector  $\vec{A}$ .



Remember:  
 $\vec{A} + \vec{B} = \vec{B} + \vec{A}$  commutative

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

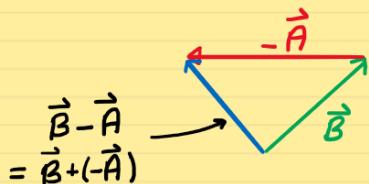
subtraction is NOT  
commutative-

$$|\vec{A} - \vec{B}| = |\vec{B} - \vec{A}| \text{ but antiparallel.}$$

Now find  $\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$



Now find  $\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$



subtraction is NOT  
commutative-

$$|\vec{A} - \vec{B}| = |\vec{B} - \vec{A}| \text{ but antiparallel.}$$

NOTE:  $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$   
 instead  $\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$

i.e.  $\vec{A} - \vec{B}$  is the negative of  
 $\vec{B} - \vec{A}$ .

### Multiplication of a vector by a scalar.

Let  $|\vec{A}| = 1m$  along the positive x-direction.

i) sketch  $\vec{A}$ .

$$\overrightarrow{\vec{A}}$$

ii) sketch  $\vec{B} = 2\vec{A}$

this means  $|\vec{B}| = 2|\vec{A}| = 2m$  and  $\vec{B}$  is parallel to  $\vec{A}$ .

$$\overrightarrow{\vec{B}} = 2\overrightarrow{\vec{A}}$$

iii)  $\vec{C} = \frac{1}{2}\vec{A}$

$\therefore |\vec{C}| = \frac{1}{2}|\vec{A}|$  and  $\vec{C}$  is parallel to  $\vec{A}$ .

$$= \frac{1}{2} m$$

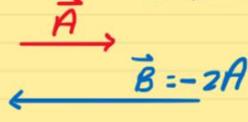
$$\rightarrow \overrightarrow{\vec{C}} = \frac{1}{2}\overrightarrow{\vec{A}}$$

$$= \frac{1}{2} m$$

$$\rightarrow \vec{c} = \frac{1}{2} \vec{A}$$

iv)  $\vec{D} = -2\vec{A}$  But  $\vec{D}$  is anti-parallel (opposite) to  $\vec{A}$

$$\therefore |\vec{D}| = 2 |\vec{A}| = 2m$$



Newton's 2<sup>nd</sup> Law

$$\vec{F} = m \vec{a}$$

$$\vec{a} = \frac{1}{m} \vec{F}$$

$\uparrow$  mass  $\propto$  parallel

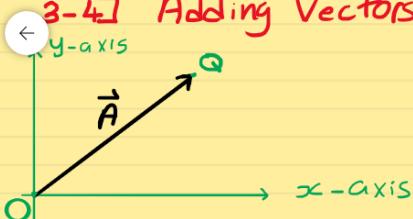
$\vec{a}$  ALWAYS  $\parallel$  to  $\vec{F}$

v)  $\vec{E} = -\frac{1}{2} \vec{A}$

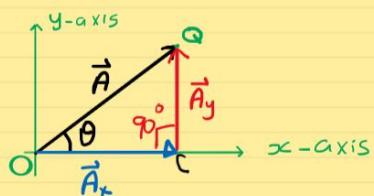
$$\vec{a} = \frac{1}{m} \vec{F}$$

$\uparrow$  mass  $\propto$  parallel

### 3-4] Adding Vectors by Components



suppose a car moved from the origin to point Q, a displacement  $\vec{A}$ . Is there an alternative route by moving along the positive x and y axes? The answer is YES.



Start from the origin:

- move a displacement  $\vec{A}_x$  along the positive x-axis.
- turn and make a displacement  $\vec{A}_y$  along the positive y-axis

$\vec{A}$  is the resultant displacement of  $\vec{A}_x$  and  $\vec{A}_y$ .

$$\therefore \vec{A} = \vec{A}_x + \vec{A}_y$$

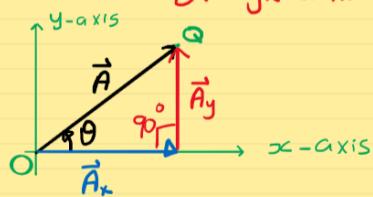
$|\vec{A}_x| = A_x$  is called the x-component of  $\vec{A}$ .

$|\vec{A}_y| = A_y$  is called the y-component of  $\vec{A}$ .

$|\vec{A}_x| = A_x$  is called the  $x$ -component of  $\vec{A}$ .

$|\vec{A}_y| = A_y$  is called the  $y$ -component of  $\vec{A}$ .

$\theta$ : angle with +ve  $x$ -axis in an anticlockwise direction.



$A, A_x$  and  $A_y$  form a right-angle triangle  $\Rightarrow$

$$A^2 = A_x^2 + A_y^2 \quad (\text{Pythagoras' theorem})$$

Also  $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \Rightarrow \cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta \quad - \textcircled{1}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \Rightarrow \sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta \quad - \textcircled{2}$

Also  $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \Rightarrow \cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta \quad - \textcircled{1}$

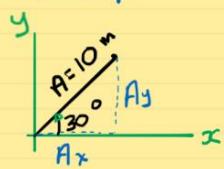
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \Rightarrow \sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta \quad - \textcircled{2}$

$\textcircled{2}/\textcircled{1} \Rightarrow \tan\theta = \frac{A_y}{A_x} \xrightarrow{\text{opposite}} \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \xrightarrow{\text{tan inverse}}$

Also  $\textcircled{1}^2 + \textcircled{2}^2 \Rightarrow A_x^2 + A_y^2 = A^2 \underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} = A^2$  as before.

If we know  $A_x$  and  $A_y \Rightarrow$  can calculate  $A$  and  $\theta$  and vice versa. ( $A, \theta \Rightarrow A_x, A_y$ ) .

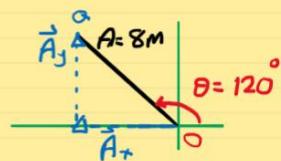
### Examples



$$A_x = A \cos 30^\circ = 10 \left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3} \text{ m.}$$

$$A_y = A \sin 30^\circ = 10 \left(\frac{1}{2}\right) = 5 \text{ m.}$$

Example



Second quadrant

along -ve x-direction

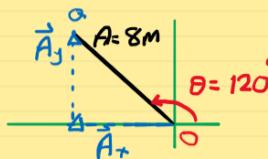
$$A_x = 8 \cos 120^\circ = 8 \left(-\frac{1}{2}\right) = -4 \text{ m}$$

$$A_y = 8 \sin 120^\circ = 8 \left(\frac{\sqrt{3}}{2}\right) = +4\sqrt{3} \text{ m.}$$

First quadrant

along positive y-direction.

Example



Second quadrant

$$\begin{aligned} \sin \theta + &\rightarrow A_y + \\ \cos \theta - &\rightarrow A_x - \end{aligned}$$

along -ve x-direction

$$A_x = 8 \cos 120^\circ = 8 \left(-\frac{1}{2}\right) = -4 \text{ m}$$

$$A_y = 8 \sin 120^\circ = 8 \left(\frac{\sqrt{3}}{2}\right) = +4\sqrt{3} \text{ m.}$$

First quadrant

along positive y-direction.

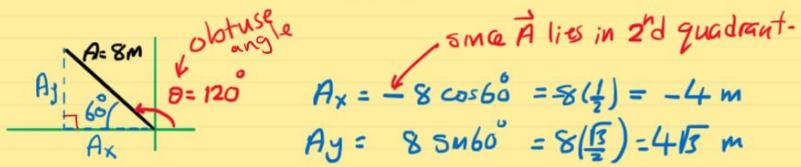
$$\begin{aligned} \sin \theta - &\rightarrow A_y - \\ \cos \theta - &\rightarrow A_x - \end{aligned}$$

Third quadrant

$$\begin{aligned} \sin \theta + &\rightarrow A_y + \\ \cos \theta + &\rightarrow A_x + \end{aligned}$$

Fourth quadrant

Alternatively: use the acute angle of  $60^\circ$ .



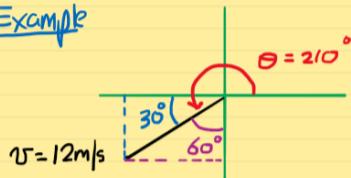
$$A_x = -8 \cos 60^\circ = -8 \left(\frac{1}{2}\right) = -4 \text{ m}$$

$$A_y = 8 \sin 60^\circ = 8 \left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3} \text{ m}$$

**NOTE:** When we use  $\theta$  with positive x-axis in an anticlockwise direction  $\Rightarrow$  the sign of each component comes out of the angle automatically.

When we use the acute angle  $\Rightarrow$  we must insert the sign of each component by hand depending on the quadrant where the vector is.

Example



Three possible ways to find  $v_x$  and  $v_y$ .

$$\textcircled{1} \quad v_x = 12 \cos 210^\circ = 12 \left(-\frac{\sqrt{3}}{2}\right) = -6\sqrt{3} \text{ m/s}$$

$$v_y = 12 \sin 210^\circ = 12 \left(-\frac{1}{2}\right) = -6 \text{ m/s}$$

**③** take right-angle triangle with  $60^\circ$  acute angle

$$v_x = -12 \sin 60^\circ = -12 \left(\frac{\sqrt{3}}{2}\right) = -6\sqrt{3} \text{ m}$$

$$v_y = -12 \cos 60^\circ = -12 \left(\frac{1}{2}\right) = -6 \text{ m}$$

