

- Conservative forces:

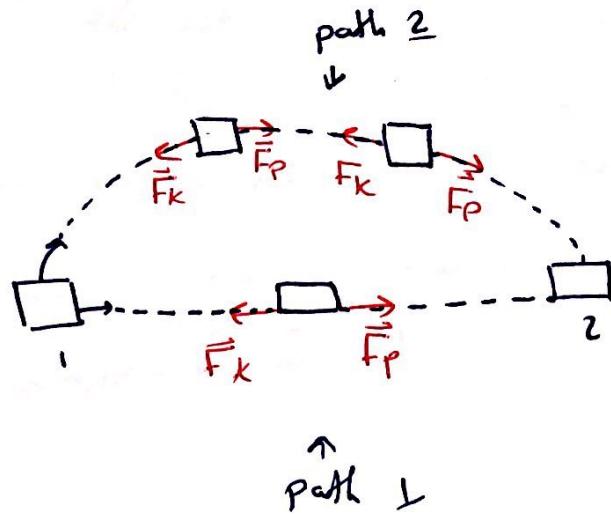
- Their work does not depend on the path.
- Their work round a closed path is zero.

- Examples of conservative forces:

- Gravitational force \rightarrow (Required in this course)
- Spring force \rightarrow (Not required in this course)
- electric force \rightarrow (Not required in this course)

- Non-conservative Forces:

- Their work depends on the path.
- Their work round a closed path $\neq 0$



- W_{fk} along the path 1 \neq W_{fk} along path 2
 ↳ work done by friction.

W_{fk} from 1 \rightarrow 2 back to 1 is NOT 0.

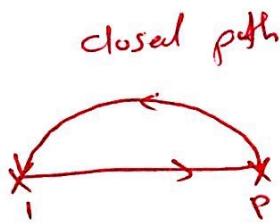
- When you raise an object up to a height h above the ground, you do work against gravity. This work is stored as gravitational potential energy in the system that consists of the object - earth system.
- What is the evidence that energy is stored?
 ↳ when you release the object it falls back to the ground.

In the figure above, work is done by the force F_p against the force of friction to move the box from point 1 \rightarrow point 3. But this energy is NOT stored. it is lost as heat for example. The box does NOT move back from point 1 \rightarrow point 2 when F_p is removed.

Example: An object acted on by a constant force (F). moves from point 1 to point 2 and back again. The work done by this force (F) in this closed trip is 60 J.

Can you determine from this info. if F is a conservative force?

Answer: F is not a conservative force since its work round a closed path $\neq 0$



* Work - Energy Extended:

$$W_{\text{net}} = \Delta K$$

$$W_{\text{nc}} + W_c = \Delta K$$

$$\text{But } W_c = -\Delta U$$

W_{net} : work done on the object by ALL forces acting on it (conservative and nonconservative).

W_{nc} : work done on object by nonconservative forces only.

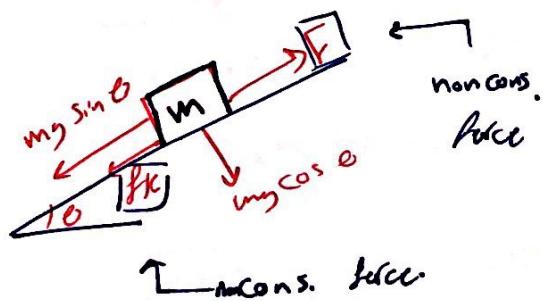
W_c : work done on object by conservative forces only.

$$\therefore W_{\text{nc}} - \Delta U = \Delta K$$

$$W_{\text{nc}} = \Delta U + \Delta K$$

$$\boxed{\therefore \Delta U + \Delta K = W_{\text{nc}}}$$

- Change in potential energy + change in kinetic energy = work done by noncons. force



6-6] Conservation of total mechanical energy

Total mechanical energy E is defined as:

$$E = K + U$$

↑ ↑ ↑
 total kinetic potential
 mechanical energy energy

If no nonconservative forces are present (e.g. free fall)

\Rightarrow No work is done by nonconservative forces.

$$\Delta K + \Delta U = 0$$

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$K_f + U_f - (K_i + U_i) = 0$$

$$E_f - E_i = 0 \Rightarrow$$

$$E_f = E_i = \text{constant}$$

i.e. conserved

$W_{nc} = 0$ since we have
no nonconservative
forces acting.

h

- $v_i = 0$
- $U_i = mgh$, $K_i = 0$
- $E_i = mgh + 0 = mgh$

v_f

$U_f = 0$, $K_f = \frac{1}{2}mv_f^2$

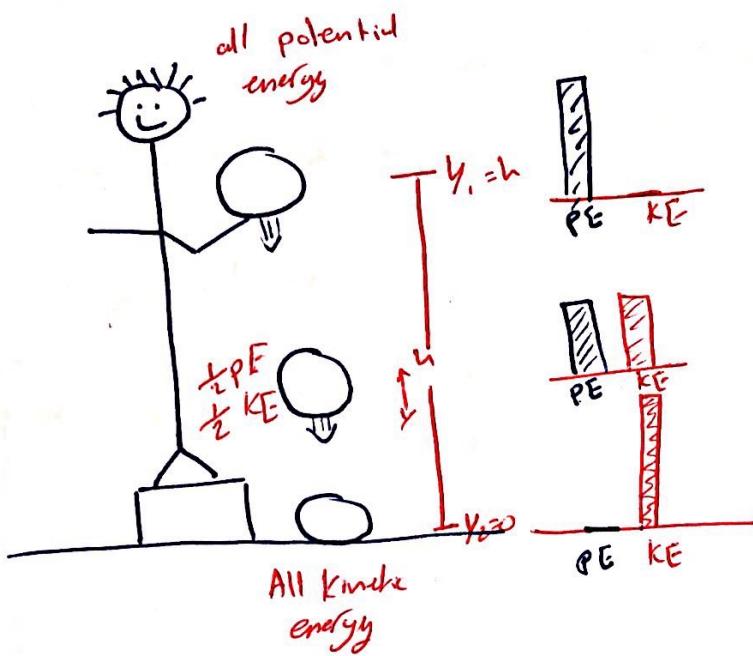
$E_f = \frac{1}{2}mv_f^2 + 0$

$E_f = E_i \Rightarrow mgh = \frac{1}{2}mv_f^2$

\Rightarrow There is No loss of total mechanical energy $E \Rightarrow$

Total mechanical energy is conserved.

$$E_f - E_i = \Delta E = 0 \quad (\text{when No nonconservative forces are present}).$$



$$PE \equiv U, KE \equiv K$$

Since ONLY conservative forces act \Rightarrow total mechanical energy is conserved i.e.

$$E = K + U = \text{constant}$$

As the ball falls only the gravitational force (mg) which is a conservative force acts $\Rightarrow U$ decreases while K increases.

But $K + U = E$ is constant.

$$E = K + U = \frac{1}{2}mv^2 + mgy$$

$$\Delta K + \Delta U = 0$$

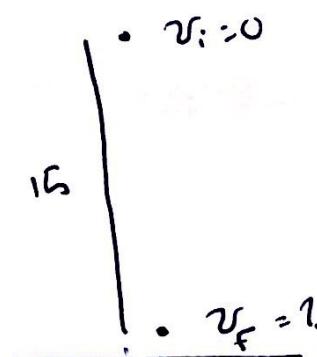
Example: An object of mass m is dropped from a height of 15m above the earth's surface. Ignoring air resistance, find:

(ii) Its speed just before hitting the ground.

Only the weight does work. It is a conservative force \Rightarrow total mechanical energy

$$\Delta K + \Delta U = 0$$

For ΔU :- If object rises $\Rightarrow \uparrow u$
 $\checkmark \Delta U = +mgh$
- If object descends $\Rightarrow \downarrow u$
(falls)
 $\checkmark \Delta U = -mgh$
- If object remains \Rightarrow —
on the same level
 $\checkmark \Delta U = 0$



$$\Delta K + \Delta U = 0$$

$$\cancel{\frac{1}{2}mv_i^2} - \cancel{mv_i^2} - mg(15) = 0$$

$$\therefore v_f^2 = 15g$$

$$\therefore v_f = \sqrt{30g} \sim 17.1 \text{ m/s}$$

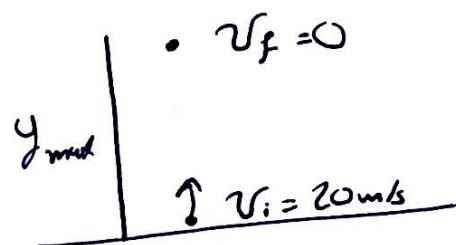
Example: A stone is projected vertically upwards from ground level with an initial speed of 20 m/s. Ignoring air resistance, find its maximum height.

only the gravitational force acts \Rightarrow

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}m(0 - (20)^2) + mg y_{\max} = 0$$

$$y_{\max} = \frac{(20)^2}{2g} = 20 \text{ m}$$



Example: Find the speed of the m₁ when it has fallen a vertical distance of 2m. All surfaces are smooth.

Assume system started from rest.

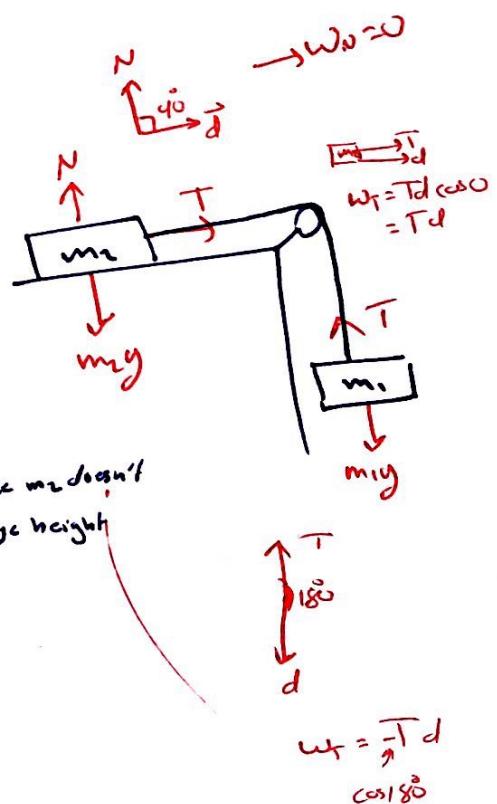
$$\Delta K + \Delta U = 0$$

$$\frac{(\Delta K_1 + \Delta U_1) + (\Delta K_2 + \Delta U_2)}{\text{for mass } 2} = 0$$

$$\frac{1}{2}m_1(v_{1f}^2 - v_{1i}^2) - m_1g(2) + \frac{1}{2}m_2(v_{2f}^2 - v_{2i}^2) + 0 = 0$$

↑ since m₂ doesn't change height

$$\frac{1}{2}m_1(v_{1f}^2 - 0) - 2m_1g + \frac{1}{2}m_2v_{2f}^2 = 0$$



m₁ and m₂ are connected by an inextensible string $\Rightarrow v_{1f} = v_{2f} = v_f$

$$\frac{1}{2}(m_1 + m_2)v_f^2 = 2m_1g$$

$$\therefore v_f^2 = \frac{4m_1g}{m_1 + m_2} \Rightarrow v_f = \sqrt{\frac{4m_1g}{m_1 + m_2}}$$

NOTE: T is a nonconservative force. But the work done by the vertical tension T is negative.

(T opposite to the downward displacement of m_1)

$$m_1 \text{ i.e. } W_T^{\text{vertical}} = Td (\cos 180 = -Td)$$

While the tension in the horizontal spring does positive work as it is parallel to the displacement of m_2 which is to right $W_T^{\text{horizontal}} = Td \cos 0 = Td$

\therefore Total work done by the tensions is $Td + (-Td) = 0$

$$\Delta K + \Delta U = 0$$